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THE

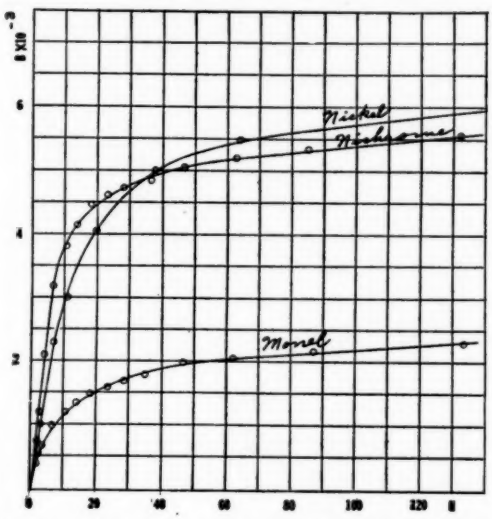
PHYSICAL REVIEW.

THE HALL EFFECT AND SOME ALLIED EFFECTS.

By Alpheus W. Smith.

The Hall Effect in Magnetic Alloys.

In an earlier paper[1] the author has examined the influence of temperature on the Hall effect in iron, nickel and cobalt. The results on these metals suggested a similar study of this effect in some magnetic alloys. For this purpose the two alloys, monel and nichrome, and three silicon steels were chosen. Monel is an alloy containing about 68 per cent. nickel, 1.5 per cent. iron, 1 per cent. manganese, and 29.5 per cent. copper. Nichrome is an alloy of nickel, iron, chromium and manganese.



Magnetisation Curves for Monel, Nichrome and Nickel.

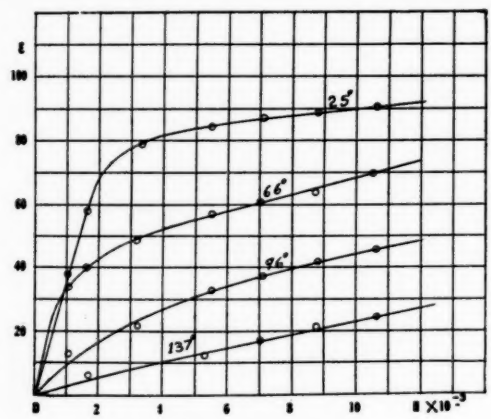
Fig. 1.

The magnetization curves for these two alloys together with the magnetization curve for nickel are given in Fig. 1. The latter curve is taken

[1] Phys. Rev., 30, pp. 1–34, 1910.

from the work of Honda and Shimizu.[1] Under corresponding conditions the permeability of monel is about one half that of nickel. For fields less than 30 dynes the permeability of nichrome is greater than that of nickel but for greater fields the opposite is true. The three steels which were examined will be referred to as Steel I., Steel II. and Steel III. Steel I. and Steel II. were high silicon steels. The former contained 3.80 per cent. silicon, 0.025 per cent. carbon, 0.010 per cent. manganese, 0.06 per cent. phosphorus, 0.025 per cent. sulphur, and 0.05 per cent. aluminum. An analysis of Steel II. was not made, but it contained about the same amount of silicon as Steel I. Steel III. contained 0.25 per cent. silicon and the same amounts of the other elements which were present in Steel I. From these alloys plates of the usual form were cut for the study of the Hall effect. These plates were about 1.6 cm. wide and 3.5 cm. long. The thicknesses of the plates were as follows: Steel I., 0.0361 cm.; Steel II., 0.0332 cm.; Steel III., 0.0351 cm.; nichrome, 0.0830 cm.; and monel, 0.0625 cm.

The apparatus differed only in minor details from that used in the former paper and the method of procedure was essentially that used in



Hall Effect in Monel.

Fig. 2.

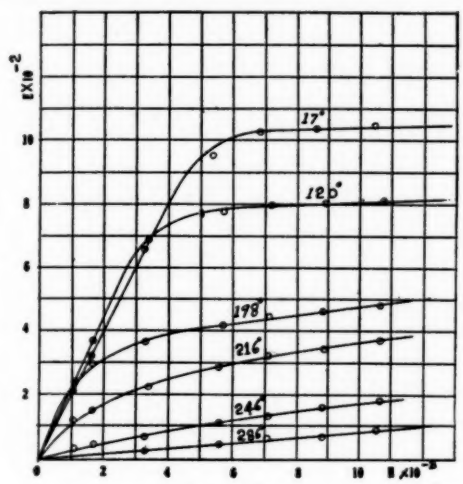
the study of the Hall effect in nickel, iron and cobalt. For the details of the apparatus and the method of procedure reference is made to the paper already cited.

In Fig. 2 have been plotted the Hall electromotive forces in monel for different magnetic fields for four temperatures. Fig. 3 contains the corresponding results for nichrome at six different temperatures. Fig. 4 shows the behavior of the Hall electromotive force in the steels at room

[1] Phil. Mag. (6), 10, p. 553, 1905.

temperature. For the sake of comparison the case of iron and that of nichrome have been included. In each of these figures the Hall electromotive forces which have been plotted are the electromotive forces which would be observed if a current of one absolute unit flowed in a plate one centimeter in thickness. The magnetic fields are in absolute units and the temperatures in degrees Centigrade.

In either monel or nichrome at room temperature the Hall electromotive force is at first proportional to the intensity of the magnetic field. When saturation begins to be reached this proportionality fails. After the characteristic knee in the curve has been passed the Hall electromotive force increases slowly with further increase in the magnetic field. Saturation occurs earlier in monel than in nichrome. An increase in temperature causes a decrease in the Hall effect except that in monel

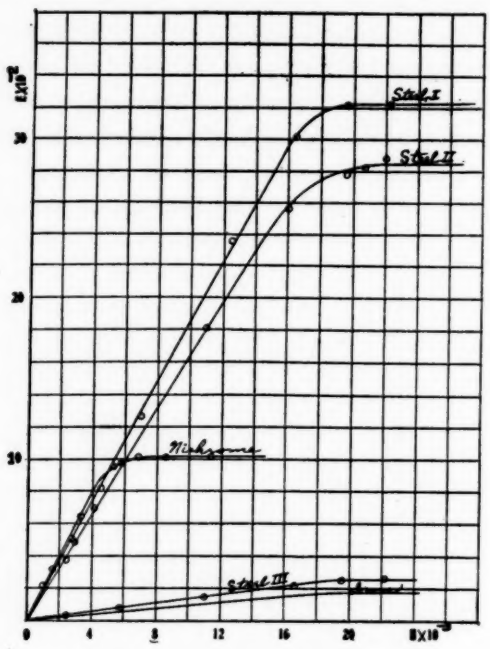


Hall Effect in Nichrome.

Fig. 3.

for fields less than 1,000 C.G.S. units there is a small increase between 25° C. and 66° C. and in the case of nichrome there is for fields less than about 3,000 C.G.S. units a small increase between 17° C. and 120° C. When the temperature of the monel has reached 137° C. and that of the nichrome 286° C. the Hall electromotive force becomes proportional to the magnetic field as in the non-magnetic metals. At room temperature the Hall effect in monel is about three times its value in nickel and its direction is the same in the two cases. In nichrome it is about sixteen times as large as in nickel and its direction is opposite to that in nickel. Fig. 4 shows the influence of silicon on the Hall effect in iron. The curve for iron is taken from an earlier paper. Steel III., which contained 0.25

per cent. of silicon, has a Hall effect about 75 per cent. greater than that in electrolytic iron and about 10 per cent. greater than that in Kahlbaum iron. The further addition of silicon causes a rapid increase in the Hall effect so that Steel I. which contained 3.80 per cent. silicon gives a Hall effect about sixteen times that in Kahlbaum iron. On the other hand the addition of silicon to the iron causes a decrease in the temperature



Hall Effect in Silicon Steels.

Fig. 4.

coefficient of the Hall effect. The direction of the Hall effect in each of these steels is the same as its direction in pure iron. Table I. gives the values of R calculated from the equation,

$$E = \frac{RHI}{d},$$

where E, H, and I are in absolute units and d in centimeters. In this table is also included the thermoelectromotive forces against copper. The first addition of silicon causes a decrease in the thermoelectromotive force and further addition of silicon causes a reversal. With this reversal of the thermoelectromotive force there is not associated a reversal of the Hall effect.

TABLE I.

Name.	Temp. °C.	H.	$R \times 10^3$.	Thermo-electric Height 0°-100°.
Monel	25°	1,650	− 35.1	
	66°	1,650	24.2	
	96°	3,150	9.62	$+22.9 \times 10^{-6}$
	137°	5,300	2.38	
Nichrome	17°	3,300	+199	
	120°	3,300	209	
	198°	1,050	200	
	216°	1,050	113	$+ 0.699 \times 10^{-6}$
	246°	3,300	21.2	
	286°	5,550	8.6	
Steel I.	19°	6,000	+152	
	130°	6,000	175	
	244°	6,000	208	$+12.4 \times 10^{-6}$
	310°	6,000	226	
Steel II.	19°	6,000	+151	
	128°	6,000	171	
	247°	6,000	199	$+12.67 \times 10^{-6}$
	311°	6,000	218	
Steel III.	18°	7,100	+ 11.5	
	129°	7,100	23.4	
	235°	7,100	44.3	$- 6.54 \times 10^{-6}$
	307°	7,100	62.3	
Kahlbaum iron	17°	6,000	+ 10.8	
	132°	6,000	22.4	
	250°	6,000	42.4	$- 9.71 \times 10^{-6}$
	321°	6,000	61.2	

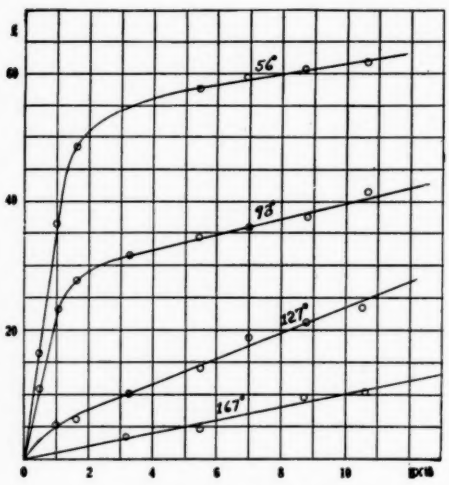
NERNST EFFECT IN MAGNETIC ALLOYS.

A study of the variation of the Nernst effect with temperature has already been made in nickel and cobalt. It was also desirable to have data on the Nernst effect in the alloys which have been studied in the earlier part of this paper with respect to the Hall effect. The thickness of the monel was 0.148 cm. The other plates were those used in the study of the Hall effect. The apparatus was essentially that used for the determination of the Nernst effect in nickel and cobalt, so that reference will be made to that paper[1] for the details of the experiment.

In Fig. 5 the Nernst electromotive forces in monel have been plotted against the magnetic fields; in Fig. 6, the corresponding values for nichrome. In Fig. 7 have been given the Nernst electromotive forces in the silicon steels when one end of the plate was at about 20° C. and the

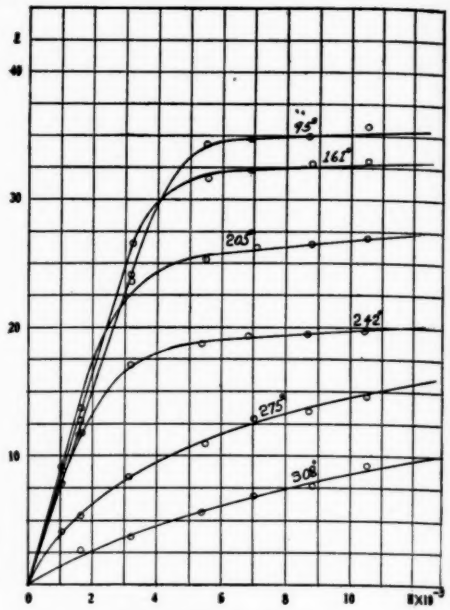
[1] PHYS. REV., 33, pp. 295-306, 1911.

other at the temperature of steam. The electromotive forces plotted for abscissæ in each of these figures have been reduced to the case that the flow of heat occurs in a plate one centimeter wide with a temperature gradient of 1° C. per centimeter in it. These curves are very similar to the corresponding curves for the Hall effect. There is at the low tem-



Nernst Effect in Monel.

Fig. 5.

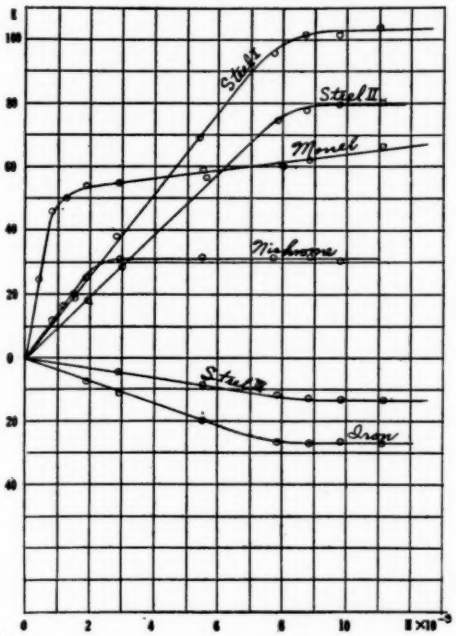


Nernst Effect in Nichrome.

Fig. 6.

peratures the initial proportionality between the electromotive force and the magnetic field with the characteristic bend in the region of saturation and then the gradual increase in the electromotive force with further increase in the magnetic field. Also a rise in temperature causes a decrease in Nernst effect except in the case of nichrome where for fields less than 3,000 C.G.S. units it increases slightly between 95° and 205° C. This is essentially the exception noted for the Hall effect. In the neighborhood of 167° C. for monel and 300° C. for nichrome these alloys seem to become non-magnetic and the Nernst electromotive force becomes proportional to the magnetic field. The direction of the effect in monel, nickel and nichrome is the same. From Fig. 7 it appears that steel containing 0.25 per cent. of silicon has a Nernst effect which is about one half of that in pure iron. The direction of the effects in the two cases is the same. The Nernst effect in steel containing 3.8 per cent. of silicon is opposite in direction to that in iron and about three and one half times as large. The addition of small amounts of silicon to the iron

causes the Nernst effect to decrease. The addition of larger amounts causes the effect to reverse its direction and to rise to a value much larger than that in pure iron. In the case of the Hall effect the addition of silicon to the iron caused an increase in the Hall effect but not a reversal of sign. The reversal of the thermoelectromotive forces occurs



Nernst Effect in Silicon Steels.

Fig. 7.

under the conditions under which this reversal of the Nernst effect occurs. If the Nernst effect is regarded as the rotation of the thermoelectromotive force in the plate the Nernst effect and the thermoelectromotive force should reverse simultaneously. This is exactly what happens in these alloys.

Table II. contains the values of Q calculated from the equation,

$$E = Q\beta H\frac{\partial T}{\partial x},$$

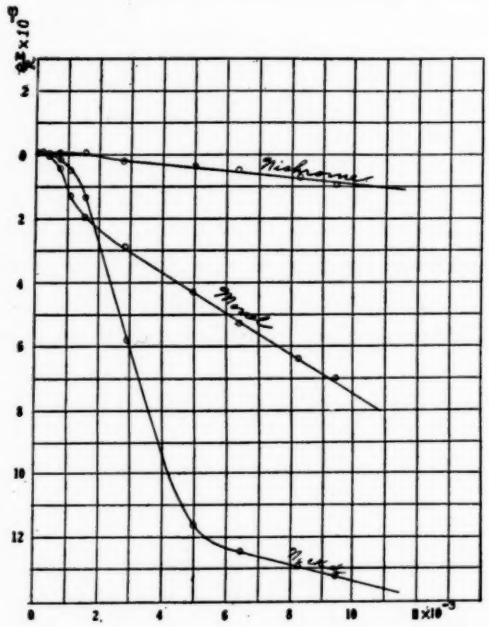
where E is the electromotive force in absolute units; H, the magnetic field in absolute units; β, the width of the plate in centimeters, and $\frac{\partial T}{\partial x}$, the temperature gradient in degrees Centigrade per centimeter.

TABLE II.

Name.	Temp. °C.	H.	$Q \times 10^3$.	Thermo-electric Heights 0°–100°.
Monel	56°	1,500	−32.3	
	93°	1,050	22.1	$+22.9 \times 10^{-6}$
	127°	1,000	5.3	
	167°	1,060	.97	
Nichrome	43°	2,500	− 7.08	
	95°	3,200	7.53	
	161°	3,250	8.12	
	205°	1,650	7.70	$+ 0.699 \times 10^{-6}$
	242°	1,650	7.00	
	275°	1,650	3.27	
	308°	7,000	.99	
Steel I	38°	15,400	− 6.19	$+12.4 \times 10^{-6}$
Steel II	33°	14,000	− 4.85	$+12.67 \times 10^{-6}$
Steel III	41°	15,600	+ 0.72	$- 6.54 \times 10^{-6}$
Iron	47°	15,600	+ 1.71	$- 9.71 \times 10^{-6}$

TRANSVERSE CHANGE OF RESISTANCE IN A MAGNETIC FIELD.

The wires were wound on thin sheets of mica as nearly as possible plane. The nickel wire which was from Hartmann and Braun was 0.030 mm. in diameter. The monel and nichrome wires were 0.075 mm. in



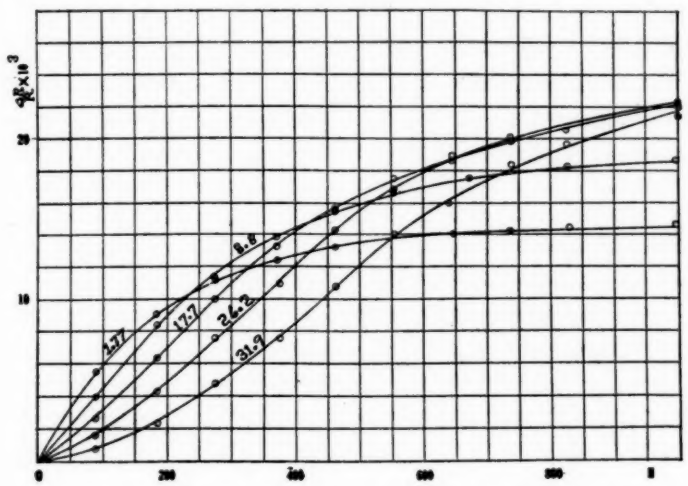
Transverse Change of Resistance in Monel, Nichrome and Nickel.

Fig. 8.

diameter. The coils were placed between the poles of a large electromagnet so that the lines of force were as nearly as possible normal to the plane of the coil. Necessary precautions were taken concerning temperature equilibrium. The observations on three coils at room temperature are recorded in Fig. 8. The slight increase in resistance for lower fields has been attributed to the failure to get the coil perfectly plane and exactly perpendicular to the lines of force so that there is superposed on the transverse change of resistance a longitudinal change arising from the component of the field parallel to the plane of the coil. The longitudinal change of resistance is an increase and for these small fields exceeds the transverse change. The transverse change of resistance in nichrome is small compared to that in nickel, and that in monel for fields greater than about 3,000 C.G.S. units is roughly one half the change in nickel.

INFLUENCE OF TENSION ON THE LONGITUDINAL CHANGE OF RESISTANCE.

It has been shown by Williams[1] that nickel wires sustaining different tensions give different values for the longitudinal change of resistance in a magnetic field. A similar series of observations has been made on



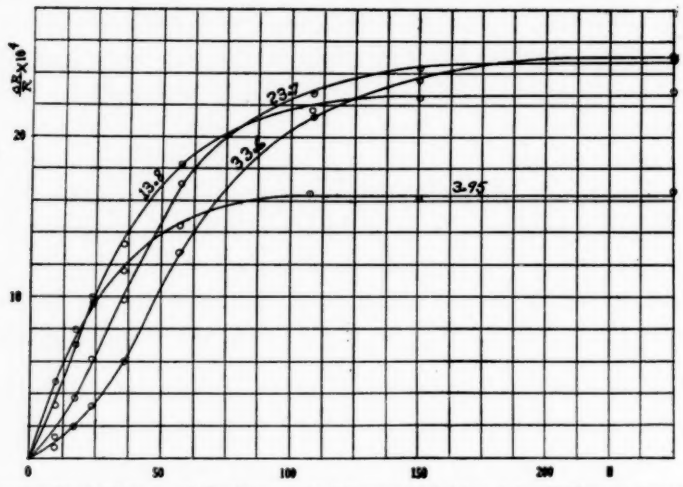
Longitudinal Change of Resistance in Nickel for Different Loads.

Fig. 9.

monel and for the sake of comparison nickel has been again studied. The monel wire was hung so as to coincide with the axis of a large vertical solenoid about one meter in length. The monel wire was about 80 cm. long and 0.0254 cm. in diameter. The solenoid was provided with a water jacket between the magnetizing coil and the space containing the

[1] Phil. Mag. (6), 6, p. 693, 1903.

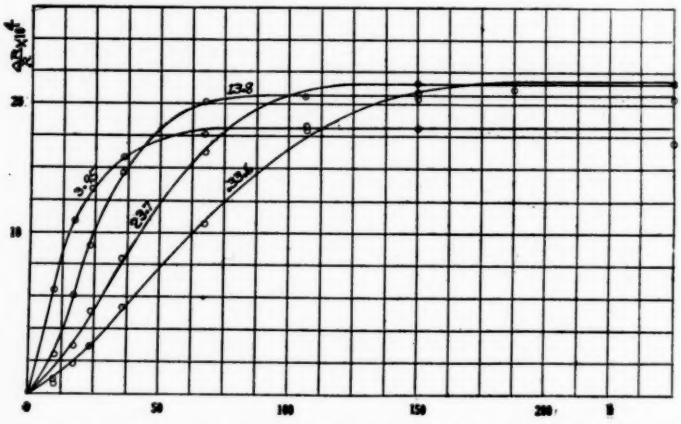
wire to be studied. Further insulation from temperature changes was afforded by filling the space between the inner wall of the water jacket and the wire with cotton wool. The tensions were obtained by hanging



Longitudinal Change of Resistance in Unannealed Monel for Different Loads.

Fig. 10.

different weights on the wire. The resistances of the wire at room temperature for different fields and different loads were measured. The wire was demagnetized after each observation. The tensions expressed in kilograms per square millimeter have been indicated on the curves in



Longitudinal Change of Resistance in Annealed Monel for Different Loads.

Fig. 11.

the figures. For the study of the nickel wire a more powerful solenoid was used. It was 46 cm. long. The nickel wire was .0425 cm. in diameter and 25 cm. long. In order to have a greater length of wire two

wires parallel to each other and near together were joined in series by soldering the lower end of each one to a heavy strip of copper to which the weights affording the tension were attached. These wires were then placed in the solenoid so that the axis of the solenoid was between them. Fig. 9 gives the results for an annealed nickel wire; Fig. 10, for the monel wire before annealing; and Fig. 11, for the monel wire after annealing. For small tensions the change of resistance rises rapidly to its limiting value beyond which it does not increase with further increase in the magnetic field. The increase in the tension makes this rise to the limiting value less rapid and at first increases the magnitude of the limiting value but for tensions exceeding a certain value of the change of resistance for the larger magnetic fields seems to be the same whatever the tension. Annealing the monel makes the influence of tension greater at lower fields and less at larger fields. The results for monel are exactly similar to those for nickel except that smaller fields are required to reach the limiting value in monel than in nickel.

INFLUENCE OF OCCLUDED HYDROGEN ON THE HALL EFFECT IN PALLADIUM.

It is well known that palladium when made the cathode is an electrolytic cell in which the positive current is carried by hydrogen occludes large quantities of hydrogen, so that when saturation has been reached the palladium is found to contain a quantity of hydrogen which at atmospheric pressure would be equal to more than one thousand times its own volume. A number of observers have shown that this occluded hydrogen causes the electrical resistance[1] to increase. This increase of resistance is approximately proportional to the quantity of hydrogen occluded. When saturation is reached the resistance is about 68 per cent. greater than that in palladium free from hydrogen.

In order to study the Hall effect in palladium which contained occluded hydrogen, plates 1.2 cm. wide and 2.5 cm. long were cut from a sheet of palladium 0.03 cm. thick. From the middle of either side of the plate projected arms to which were soldered or welded palladium wires which lead to the galvanometer leads. These Hall electrodes were adjusted so as to be nearly on the same equipotential. To the ends of the plates were soldered heavy strips of copper which served as primary electrodes. The plate was mounted on a sheet of mica from which a rectangular opening had been cut so that both sides of the plate were exposed. The leads to the plate were encased in small rubber tubing and the copper parts covered with wax to protect them from the acid. The plate was

[1] McElfresh, Proc. Amer. Acad., 39, p. 323, 1904.

then mounted on a frame midway between two platinum plates which were parallel to it. These platinum plates were much larger than the palladium plate. They were joined in multiple and served as the anode of the electrolytic cell. The electrolyte was a seven per cent. water solution of sulphuric acid. When the palladium plate was in position in this cell the current flowed from the platinum plates to either side of the palladium plate so that hydrogen would be deposited at the same rate on either of the faces. Over the palladium plate in such a way as to collect the hydrogen evolved at it, was an inverted burette to which had been joined a funnel. In series with this electrolytic cell was an ordinary hydrogen voltameter with platinum electrodes. The difference between the quantity of hydrogen generated in this voltameter and the quantity generated at the palladium electrode gave the quantity of hydrogen absorbed by the palladium.

The Hall effect was first determined in the plate in the usual way when the plate was free from hydrogen. The intensity of the magnetic field was 20,300 C.G.S. units and the observations were made at room temperature. The plate was then placed in the electrolytic cell and the current allowed to flow until the plate had absorbed about one thousand times its volume of hydrogen. The Hall effect in the plate was again determined. Two plates cut from the same sheet of palladium were examined. In the first of these, the palladium lead wires were soldered to the Hall electrodes; in the second plate they were welded. The second plate gave less trouble on account of thermoelectromotive forces, so that the results for it are more accurate than those for the first plate. Within the error of observation, which did not exceed two per cent. for the first plate and one per cent. for the second plate, there is no change in the Hall effect caused by the occlusion of the hydrogen. Table III. gives the results for the two plates.

TABLE III.

	Current in Amps.	Thickness in Cm.	R before Saturation with H.	R after Saturation with H.
Plate I.....	4.50	0.0309	7.69×10^{-4}	7.54×10^{-4}
Plate II.....	4.45	0.0305	7.54×10^{-4}	7.49×10^{-4}

In consideration of the change of electrical resistance under analogous conditions, this result was scarcely to be expected. Work on the Hall effect in alloys indicates that alloys with high electrical resistances have large Hall effects. The occlusion of hydrogen, however, does not change the electrical resistance and the Hall effect in the same manner. If the Hall effect depends on the mean free path of the electrons, the

occlusion of such a large quantity of hydrogen would be expected to change its magnitude.

HALL EFFECT AND POLYMORPHISM OF TELLURIUM.

From a study of the thermoelectromotive force and the electrical resistance of tellurium Haken[1] has concluded that tellurium exists in two crystalline forms. One of these forms seems to be stable above 350° C.; the other, stable below that temperature. It seemed that the study of the variation of the Hall effect with the temperature might give additional evidence of this polymorphism.

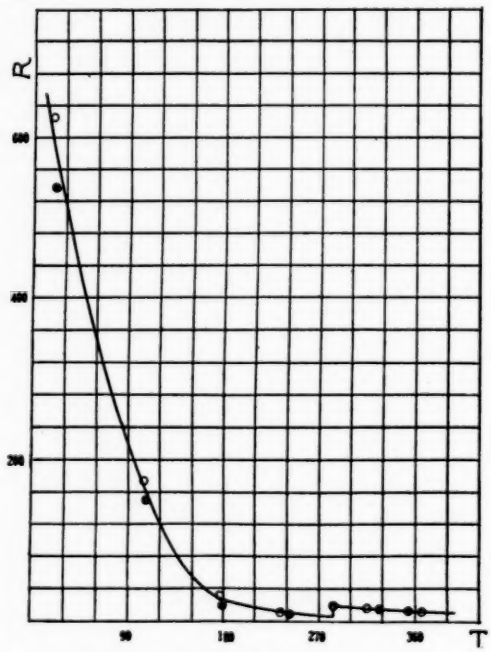
The tellurium was obtained from Kahlbaum. The plates were cast by heating the metal in a crucible and pouring it into a mould cut out of lavite. These plates were about 4 cm. long, 1.4 cm. wide and 0.3 cm. thick. From the middle of either side projected an arm about 1 cm. long and 0.4 cm. wide. These arms when fastened to the wires leading to the galvanometer served as the secondary or Hall electrodes. Either platinum or silver wires were fastened to the plate at the primary as well as at the secondary electrodes. The temperatures which were obtained by means of an electric furnace were measured with a platinum platinum-irridium thermal couple. The magnetic field had a strength of 8,800 absolute units. It was found that the way in which the plates were cooled and the heat treatment to which they were afterward subjected very largely changed the value of the Hall constant at room temperature. At higher temperatures the variations from this cause were not so large. The character of the curve showing the relation between the Hall constant and the temperature remained essentially the same whatever the heat treatment or the condition of cooling.

A number of plates were investigated. The values of the Hall constants obtained for two of these plates have been given in Table IV. and the same values have been plotted in Fig. 12. The plates for which these values are given were cast as nearly as possible in the same way by pouring the moulten tellurium into the mould at room temperature. The observations here tabulated are those made on the first heating of the plate from room temperature to the highest temperature indicated on the curve. The points on the curve marked with a circle refer to Plate I. and those marked with a circle and a cross refer to Plate I. A single curve has been drawn for both sets of points, although two curves, one lying a little above the other would better represent the facts. From this curve it is seen that with rising temperature the Hall effect in tellurium decreases very rapidly until 180° C. is reached. Between 180° and

[1] Ann. d. Phys. (4), 32, p. 291, 1910.

TABLE IV.

Plate I.		Plate II.	
Temp. C°.	R	Temp. C°.	R
21.8	536	20.3	621
109	147	105	171
180	18.8	177	30.5
243	7.47	234	8.96
283	18.0	283	19.3
327	14.9	315	18.1
353	12.0	366	12.5



Hall Effect in Tellurium.

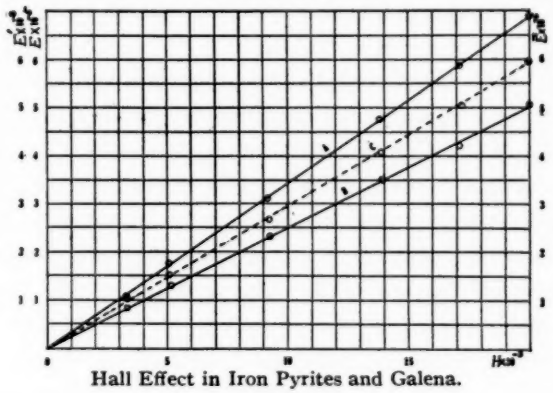
Fig. 12.

about 275° C. the effect decreases slowly. In the neighborhood of 275° C. a molecular transformation occurs in which the Hall constant is doubled. With further rise of temperature the constant decreases slowly as compared to the decrease at lower temperatures. From room temperature up to about 275° C. the tellurium is evidently a mixture of the two crystalline forms. One of these forms will be in unstable equilibrium. The relative concentrations of these crystalline forms will vary as the temperature is changed. In the neighborhood of 275° C. one of these crystalline forms goes over into the other and there remains a single crystalline form in which the Hall effect behaves as in non-magnetic

metals. These results on the Hall effect in tellurium confirm the results of Haken that tellurium exists in two crystalline forms. The temperature at which this molecular transformation occurs as indicated by the Hall effect does not agree very well with the temperature at which Haken found the behavior of the thermoelectromotive force and the electrical resistance indicated a molecular transformation.

THE HALL EFFECT IN IRON PYRITES AND GALENA.

Very little is known concerning the Hall effect in crystals. This lack of information is largely due to the difficulty in securing suitable crystals from which to cut plates adapted to the study of this effect. It was, however, found possible to obtain a plate of iron pyrites and two plates of galena in which the Hall effect could be observed. The crystal of iron pyrites was a large unmodified cube from which was cut a plate in such a way that its plane was parallel to one of the faces of the cube. This plate was 2.2 cm. long, 1.1 cm. wide and 0.18 cm. thick. Galena also crystallizes in the isometric system and each of these plates was cut so that its plane was parallel to one of the faces of the crystal. One of these plates was 1.5 cm. long, 0.8 cm. wide and .30 cm. thick; the other was 2.0 cm. long, 0.8 cm. wide and 0.303 cm. thick. The ends of the



Hall Effect in Iron Pyrites and Galena.

Fig. 13.

plates were copper plated in order to make it possible to solder wires to them for the primary electrodes. In the middle of each side of the plates a small region was also electroplated. At these points were soldered the wires which served as the secondary electrodes. The observations were made in the usual way.

Fig. 13 shows the results of these observations. In this figure the magnetic field in absolute units has been plotted for abcissæ and the electromotive force which would be observed in a plate one centimeter thick with a current of one absolute unit in it, has been plotted for ordinates.

Curve C is for iron pyrites. The ordinates for this curve are on the extreme left and have been denoted by E'. Curve A is for the first specimen of galena and curve B for the second specimen of galena. The ordinates of the former denoted by E are on the left; the ordinates of the latter on the right. These curves show that the Hall electromotive force for iron pyrites and galena is proportional to the magnetic field. At room temperature $R = -0.296$ for iron pyrites; $R = -34.2$ for first specimen of galena; and $R = -251$ for second specimen of galena. The negative sign means that the effect in these crystals has the same direction it has in bismuth. These crystals like bismuth are thermoelectrically negative with respect to copper. From the approximate relation between the thermoelectric series and the Hall effects, the Hall constant in these crystals would be expected to have the sign it has in bismuth.

SUMMARY.

1. An increase of temperature causes changes in the Hall effect and in the Nernst effect in monel and nichrome which are very similar to the changes produced in nickel, iron and cobalt under corresponding conditions.

2. The Nernst effect and the Hall effect in monel and in nichrome depend on the temperature in essentially the same way.

3. The addition of small quantities of silicon to iron causes a large increase in the Hall effect and a decrease in its temperature coefficient. Under similar conditions there is at first a decrease in the Nernst effect, then a reversal of its direction and an increase to a larger value in the opposite direction. Associated with this reversal of the Nernst effect is the reversal of the thermoelectromotive force against copper.

4. In a transverse magnetic field the resistance of monel and nichrome behaves like the resistance of nickel except that the changes under corresponding conditions are less than in nickel.

5. The influence of tension on the longitudinal change of resistance in a magnetic field in monel and in nickel is the same except for magnitude

6. Large quantities of occluded hydrogen do not change the Hall effect in palladium.

7. In the neighborhood of 275° C. there is a molecular transformation in tellurium. The Hall effect after this transformation is about double its value before this transformation.

8. The Hall effect has been determined in iron pyrites and galena. In these crystals the Hall electromotive force is proportional to the magnetic field.

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THE EFFECT OF DIELECTRICS ON UNIPOLAR INDUCTION.

BY E. H. KENNARD.

IN a recent article[1] Professor Barnett takes issue with the conclusions which I drew[2] from an experiment of my own on unipolar induction and which I should *a fortiori* draw from his results. In the Physikalische Zeitschrift I have already replied[3] to his chief criticism as presented in an earlier issue of that magazine,[4] but a further reply in connection with his later article may not be amiss.

Professor Barnett contends that neither my experiment nor his own decides the old question as to "motion of the force lines," but his argument appears to me to be fallacious in two particulars. The difficulty in both cases arises from the fact that on the moving force line theory e (the motional intensity) does not usually satisfy Laplace's equation.

In the first place, he assumes tacitly on p. 326 and explicitly on p. 327 that the electric density is proportional to div f. This was the assumption made by H. Hertz, who identified f (total intensity) and E (electric force) and rejected the motional intensity e. But on Lorentz's theory this assumption is not always allowable, and it may easily be shown to be incompatible with the moving line theory in the exact case mentioned by Professor Barnett on p. 327.

For suppose that the force lines there rotate with the magnet and let the surrounding dielectric be free ether. Then in the ether div E must vanish. But since $e = -[vB]$, where v = velocity of force lines,

$$\text{div } e = -(B \text{ curl } v) + (v \text{ curl } B),^5$$

or since

$$\text{curl } B = 0,$$

$$\text{div } e = -2(BU), \tag{1}$$

where U = vector angular velocity of lines. Thus div e and therefore div f = div $(E + e)$ will not usually vanish even where there is no electrification. I see no escape from this conclusion except by assigning to the ether some very peculiar properties invented ad hoc.

[1] PHYS. REV., Nov., 1912, p. 324.
[2] Phil. Mag., June, 1912, p. 937.
[3] P. ZS., Dec. 1, 1912, p. 1155; Mar. 15, 1913, p. 250.
[4] P. ZS., Sept. 1, 1912, p. 803.
[5] [ ] denotes the vector product, ( ) the scalar product.

In his latest article[1] on the subject Professor Barnett refers to discussions of unipolar induction by Poincaré and M. Abraham. Careful reading shows, however, that both of these authors approach the subject from the Hertzian standpoint and are thereby led virtually to make the same assumption as Professor Barnett concerning div. f. Furthermore, both authors dismiss the question as to whether the lines "move" or not with the observation that the lines are creatures of our imagination, so that the question has no sense. But neither author so much as mentions the substantial physical theory, apparently once a favorite in Germany, which employs the "moving lines" merely as a symbol, and which differs from Lorentz's theory chiefly in that in the equation

$$E' = E + 1/c[vB]$$

v is interpreted as velocity not relative to the ether but relative to the material system which is the source of the magnetic field. Hence for our purpose both of these references are really beside the point.

In the second place, in note (1) on p. 326 Professor Barnett does not state why $K - 1/K$ is to be replaced by unity. From the article there referred to, the reason seems to be that the moving lines are supposed to act on the ether (p. 433, top) in the same manner as on a material dielectric. But such an assumption is not a part of "current theory" as I know it, and I do not see how it is to be harmonized both with electrostatics and with the moving line theory. For simplicity, let us suppose that in the case cited above the magnet is electrically uncharged. Then if, as Professor Barnett assumes,

$$D = KE + Ke = Kf$$

we have everywhere div $D = 0$, curl D = curl $e = 0$, and therefore D and f vanish at all points, together with the electricity density. Hence for electrostatic reasons div E should vanish, but it cannot do so because as shown above div e does not vanish. The only explanation in harmony with electrostatic theory would be the assumption that E itself contains a component due to the moving lines, but all such effects were included in e.

The real root of the difficulty appears to me to lie deeper yet. In the article to which the note refers Professor Barnett seems to treat the ether exactly like a material dielectric. But in electrostatics ether and matter are diametrically opposed: an outward displacement in matter (across a closed surface) leaves behind it in the matter a negative (free) charge, whereas an outward displacement in free ether leaves behind it a

[1] P. ZS., March 15, 1913, p. 251.

positive charge. And this difference is essential,—ether and matter are complementary: for if an outward displacement in matter is produced by any other cause than the insertion of a positive charge the ether must be free to take on a displacement backward corresponding to the change in E and serving to keep the total displacement unaltered.

Let us now combine with the moving line theory Lorentz's relation between D, E and e, and calculate the charge thus obtained on Professor Barnett's condenser. The result will likewise hold on this theory when the solenoid is at rest and the condenser rotates in the opposite direction, and in this case will also be true according to Lorentz's theory. We have in the short-circuiting wire $f = E + e = 0$, hence

$$E = -e = \omega B r,$$

where ω = angular velocity of lines (relative to condenser); and

$$\psi = \omega B \frac{r''^2 - r'^2}{2}, \tag{2}$$

where ψ = P.D. between inner and outer cylinders. In the dielectric

$$D = KE + (K - 1)e, \tag{3}$$

$$\therefore (rD)\frac{dr}{r} = KE dr - (K - 1)\omega B r dr,$$

and since (rD) is constant we have after integrating and applying (2)

$$D = \frac{\psi}{r \log \frac{r''}{r'}}. \tag{4}$$

Hence also

$$E = \frac{1}{K}\left\{ \frac{\psi}{r \log \frac{r''}{r'}} + (K - 1)\omega B r \right\}, \tag{5}$$

$$f = \frac{1}{K}\left\{ \frac{\psi}{r \log \frac{r''}{r'}} - \omega B r \right\}. \tag{6}$$

The last equation agrees (except with $-\omega$ for ω) with Professor Barnett's calculation on p. 326 if the factor $K - 1/K$ is there retained.

Finally if σ_1, q_1 are surface and linear charges on inner cylinder, by (4)

$$q_1 = \frac{2\pi\psi}{\log \frac{r''}{r'}}. \tag{7}$$

Since in the metal $f = 0$, $D = E = \omega Br$ and by (4)

$$\sigma_1 = \frac{\psi}{r' \log \frac{r''}{r'}} - \omega Br'.$$

If the inner cylinder is solid it contains a body charge $\rho_1 = \text{div } D$ or

$$\rho_1 = 2\omega B. \tag{9}$$

The charge on the condenser is thus independent of K, as is stated also by Professor Barnett. This result may be generalized as follows: If a rigid system composed of conductors and a homogeneous dielectric surrounded by a conducting surface, rotates about an axis of symmetry of a magnetic field (or is "cut by moving lines" which thus rotate), the resulting distribution of electrification is independent of the dielectric constant.

For under these conditions curl $f = 0$ and the line integral of f around any closed path, say $ABCDA$, where ABC lies in the dielectric and CDA in a conductor, is zero. But in the conductor $f = 0$, hence the integral of f along ABC must also be zero. Now $D = E + (K - 1)f$, hence the line integral of D along ABC equals the line integral of E, *i. e.*, equals the P.D., ψ, between A and C. But since along CDA $f = 0$, ψ equals the induced E.M.F. in CDA. Now also in the whole system curl $D = 0$ and in the dielectric div $D = 0$. Hence D is everywhere determined in terms of the induced E.M.F. in the conductors, and the electrification on the conductors is independent of the dielectric coefficient. That is, e in the dielectric merely cancels the polarization produced by the charges on the conductors.

This general theorem evidently applies to my own experiment, where a magnet rotated inside a closed conductor and an induced charge was sought (in vain) on a cylinder surrounding the latter. I think therefore that both Professor Barnett's results and my own do invalidite at least the simplest and most natural form of the old moving line theory.

Elsewhere I have called attention to the fact that Professor Barnett's own conclusion as to relative motion rests in part upon an inference. He deduces the existence of a charge in his Case I. from known laws combined with results of Blondlot, H. A. Wilson and Barnett on the effect of dielectrics moving in a magnetic field. This seems not quite conclusive because in the latter experiments sliding contacts (or a similar device) and stationary connecting wires that did not lie along the axis of the system were necessary adjuncts, and the dielectric was naturally

not enclosed in a conducting screen that rotated with it. It seems possible that such a screen might prevent the production of any charges inside it. Such an effect is indeed required by the old theory of Hertz, which is however in other respects not in accord with the results of these experiments. It would thus seem desirable that Professor Barnett's experiment, fundamental as it is, should be repeated in such a manner that both positive and negative results could be obtained with the same apparatus.

THE STABILITY OF RESIDUAL MAGNETISM.

BY N. H. WILLIAMS.

A. THEORETICAL.

I. Analogies between Elastic and Magnetic Phenomena.

THE magnetization of iron under the influence of a magnetizing field is often considered as a phenomenon analogous to the distortion of an elastic body under stress. The elastic strain theory has been built upon this analogy. The intensity of a magnetizing field at a point is defined as the force per unit pole at the point. It is a vector quantity and in a homogeneous medium is a continuous function, hence it is permissible to think of the flux of the vector per unit area perpendicular to the vector as a measure of the field intensity; thus it becomes analogous to stress in a body which is in a condition of strain. The substance in which the magnetic field is established undergoes a change which is also a vector point function and its flux per unit area is called the magnetic induction at a point, and is analogous to the strain in a body under stress. The ratio of the magnetic induction to the intensity of the magnetizing field is called the permeability of the medium and is analogous to the reciprocal of an elastic modulus.

The deformation of an elastic body occurs in approximate accordance with Hooke's law; but beside this approximate proportionality of stress and strain, other effects are observed such as elastic after-effect or hysteresis, plastic yield, permanent set due to repeated application of the load, and the increase of elastic limit with over-strain.

A magnetic phenomenon which is analogous to elastic after-effect may be observed when a demagnetizing field is applied to magnetized iron or steel. After the first partial demagnetization due to the field, there is a further slight demagnetization which is too slow to produce an appreciable effect upon a ballistic galvanometer. There is a similar time lag when the field is in such a direction as to increase the induction, and the effect is much more apparent in soft iron than in steel. These facts have been mentioned by Lord Rayleigh[1] and by Ewing.[2]

[1] Phil. Mag., March, 1887.

[2] Phil. Trans., 1885, p. 569.

Plastic yield and permanent set are analogous to the magnetic phenomena resulting from any change of magnetic induction exceeding a limit beyond which the specimen fails to return to its former state upon being removed from the field.

It was first shown by Fromme[1] and afterward by Ewing[2] that repeated applications of a magnetizing field produce a higher residual induction than a single application. This is analogous to the fact that repeated application of the load upon an elastic body may produce a permanent set provided the stress approaches the elastic limit.

II. Definition of Terms.

It will be convenient to use the term elastic limit as applying to magnetic phenomena and meaning the maximum intensity of field which may be applied to a specimen without a permanent change of its induction. A *partial* demagnetization will be designated merely as a demagnetization and the word recovery is here used to represent the change of flux which occurs when the demagnetizing field is removed.

III. Purpose of the Work.

The purpose of this paper is to present some experimental work which suggests further analogies between elastic and magnetic phenomena and especially to determine for different kinds of iron and steel the relation between the recovery and the maximum residual flux.

IV. The Scope of the Work.

The experiments show an approximate proportionality between magnetic stress and strain and that the limit beyond which the iron fails to regain its former condition after partial demagnetization may under favorable circumstances reach a variation of magnetic induction as high as 2,000 lines per sq. cm.

It is shown that for hard steel the hysteresis, for changes within the elastic limit, is quite small and the analogy between magnetic hysteresis and mechanical hysteresis is made evident by comparing this work with work recently done upon the elastic properties of steel tubes.

The effect of repeated application of a *demagnetizing* field is found to be similar to that found by Fromme and Ewing for a *magnetizing* field, *i. e.*, many repetitions of a magnetic stress, which is equal to the elastic limit, produce a change of induction that is equivalent to a permanent set.

An analogy is found for the increase of elastic limit with over-strain.

[1] Pogg. Ann., 1875; Wied. Ann., IV., 1878.

[2] Phil. Trans., 1885, p. 570.

B. EXPERIMENTAL.

I. Madelung's Experiment.

Madelung[1] has published an account of an excellent piece of work on the magnetizing effects of high frequency currents. His method involved the use of the Braun tube in plotting variation of magnetic flux in steel wire with variation of field intensity caused by the discharge current from a condenser. Fig. 1 is a copy of one of his curves showing the demagnetizing effect of an aperiodic discharge. He plots what he calls the maximum hysteresis curve, *i. e.*, the curve resulting from carrying the magnetizing field to a very high value and then reducing it. It will be observed that the curve due to the discharge lies outside the maximum

Fig. 1.

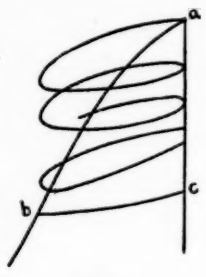


Fig. 1*a*.

curve. This is due to the effect of eddy currents in the steel, which set up a counter magnetizing field and cause a shift in phase of the actual magnetizing field. The curve obtained from the instrument gives the relation of the magnetic induction to the magnetizing field as it would be if not modified by eddy currents. The three curves of Fig. 1 were taken with wires of different size, the curve taken with small wire being the lowest one. Since the eddy current effects decrease with diminished size of wire, this lowest curve would be expected to conform more nearly to the maximum hysteresis curve than the others, as it does.

Fig. 1*a* shows the effect of repeated application of the demagnetizing impulse. The curve falls to lower values of the induction as the impulse is repeated and finally reaches a stationary position where it becomes a closed curve. However, it never falls to a position where its extreme point lies on the hysteresis curve, but the point remains outside the area of the curve. Madelung points out that this is an error due to the eddy currents and suggests that if the current were of long duration and its time rate of variation at all times small, the curve traced would be the maximum hysteresis curve down to the point *b*, and from there it would

[1] Annalen der Physik, 17, 1905, p. 861.

be the line *bC*. This is undoubtedly true for the first application of the demagnetizing field but the question arises: what would be the result of many repetitions of the demagnetizing impulse? A part of the experimental work herein described deals with this question. An effort has been made to reduce the errors due to eddy currents and end effects to a negligible magnitude and furthermore to deal with the whole matter in a quantitative way so that we may know to just what extent demagnetization is followed by recovery and how much hysteresis occurs.

II. General Method.

The method adopted in the experimental work was an oscillographic method devised by the author and so far as he is aware it has not been used elsewhere. The specimen to be studied was of steel wire in the form of a ring wound with primary and secondary coils. The secondary of a transformer operating on a sixty cycle current was connected in series with a storage battery and adjustment made so that the maximum value of the alternating E.M.F. was equal to the storage battery E.M.F. This gave a unidirectional pulsating current fluctuating between the limits of zero and a maximum value. This current was made to pass through the primary coil on the magnetized core in such a direction as to produce a demagnetizing field. A portion of this current operated one loop or vibrator of the oscillograph while the other loop was connected to the secondary consisting of a few turns and also wound on the steel core. The inductance in the secondary was negligible and hence the ordinate of the curve obtained by the oscillograph from this coil is proportional to the rate of flux variation while the ordinates of the current curve represent the instantaneous value of the magnetizing field. This latter statement is only true provided the results of eddy currents and end effects are negligible.

Proper calibration is necessary in order to obtain actual values of field intensity and rate of flux variation from these curves.

In order to obtain the curve whose ordinates represent the change of flux due to the demagnetizing field, the curve whose ordinates represent the time rate of change of flux is integrated graphically, the process being as follows: The curves as photographed by the oscillograph are transferred by means of a pantagraph to cross-section paper, the copy being drawn to double scale. The paper used was divided into twentieths of an inch. At the instant when the ordinate of the curve representing rate of flux variation is zero and increasing in the positive direction, the flux is a maximum. Directly above this point *a*, Fig. 2, at the top of the sheet a line *bc* is drawn across the quarter inch in which this point lies

parallel to the axis of abscissæ. Across the next quarter inch a line *cd* is drawn from the extremity of the preceding line and its slope is made equal to the negative of the ordinate *mn* of the curve representing time rate of flux variation at the middle point of this quarter inch. Where this line ends, another begins and crosses the next quarter inch with a negative slope equal to the ordinate of the flux rate curve at the middle of this space. The process is continued until the curve for the cycle is completed. If the work is carefully done the ordinate of the curve, measured downward, represents the flux variation with considerable accuracy. If the ordinary hysteresis curve is wanted an alternating

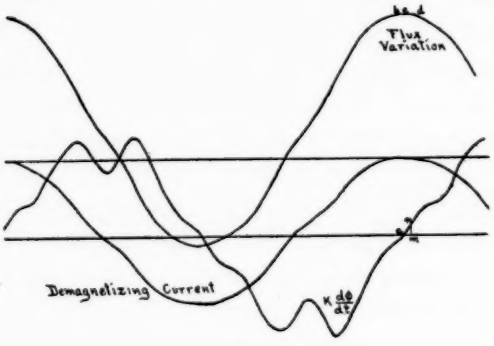
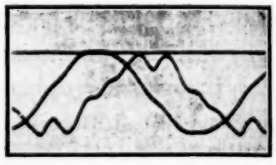


Fig. 2.



Oscillogram. The smooth curve represents the unidirectional primary current; the irregular curve the secondary current.

Fig. 2*a*.

current is used instead of the unidirectional pulsating current. The values of H, as computed from the ordinates of the current curve are plotted as abscissae and the values of the flux or the induction B, computed from the flux curve, are plotted as ordinates. Thus we have the hysteresis curve when the alternating current is used and the demagnetization and recovery curve when the pulsating current is used. A hysteresis curve obtained by this method is shown on Fig. 7, the specimen being piano wire in its normal condition.

The method has several advantages over the ballistic method for work involving small changes of B and H. It is possible to determine as many points on the curve as desired even when the curve represents very small changes. The apparatus is quite sensitive to small changes of flux while for large changes it is only necessary to reduce the number of secondary turns and put resistance into the circuit. For example, one curve was plotted in which the flux variation was only 114 lines per sq. cm. and it showed all the characteristics of the curves representing much larger changes; on the other hand hysteresis curves representing changes of magnetic induction of more than 30,000 lines per sq. cm. are easily obtained. The method also insures conditions giving a closed

curve for the cycle since the impulse is repeated a large number of times before the photograph is taken and the cycle is traversed more than 1,000 times during the exposure of the sensitive paper.

The remaining details of the method were different for different specimens and will be mentioned in connection with the experiments to which they apply.

III. Calibration.

A ballistic galvanometer was used for a part of the work. It was calibrated by means of a current inductor in the usual way.

One mm. deflection was found to correspond to a flux variation of 32.6 lines of induction.

The same current inductor was also used in calibrating the oscillograph. A wire was used for the secondary coil of such a size that its resistance per turn was the same as that of the secondaries of the steel cores. This secondary was connected to the same loop of the oscillograph that was afterward used for the secondaries of the steel cores. The maximum flux through the coils was computed from the same formula as that used in calibrating the galvanometer. Calibration curves were taken with 25, 50, 75 and 125 turns respectively in the secondary. The unidirectional current was used here as in the work upon the steel cores. The flux curve in each case was obtained by graphic integration as previously described. The maximum ordinate of this curve corresponds to the flux as computed from the maximum current. The value in terms of lines of induction of one division of the cross-section paper is thus determined, *e. g.*, with 25 turns one division corresponds to 22.9 lines of induction. Since the maximum flux variation is used in this determination, it is evident that it must be computed from the maximum current rather than from the effective value.

Since a direct current instrument having its field furnished by a permanent magnet indicates the mean current flowing through it and since the mean value of an alternating current furnished by a transformer is zero, a direct current ammeter reads one half the maximum current when the current is unidirectional and fluctuates between zero and a maximum value. The instrument used was a Weston ammeter reading directly to .02 of an ampere and having large divisions so that one could estimate to .005 ampere with accuracy.

As an illustration of the accuracy of the process, two calibration curves for a secondary of 75 turns were taken at different times, one about two weeks later than the other and under conditions giving very different wave forms. The flux value of one division of the paper was 8.104 in

one case and 8.089 in the other, showing a difference of two tenths of one per cent.

A separate calibration was made for plotting the hysteresis curves of the specimens. This became desirable since a secondary of only one turn was used and some resistance was inserted to reduce the induced current in the secondary circuit to a value that could be used with the oscillograph.

The magnetic flux per division of the cross-section paper as given by the calibration curves was as follows, the numbers in the first column being the number of secondary turns.

125 turns,	6.65 lines.
75	8.09
50	11.94
25	22.90

IV. Experiments with Specimen A.

The first specimen to be studied was of piano wire 1.03 mm. in diameter. It was in the form of a ring of mean circumference 25 cm. and cross-sectional area 1.4 sq. cm. A gap was cut in the ring about one cm. long and a plug of soft iron wire was made to fit it. The ring was heated to a red heat in an electric furnace and then plunged into cold water. The core was next insulated with tape and a secondary coil consisting of five sections of twenty-five turns each was wound upon it. The primary of 350 turns was wound outside the secondary.

The gap was made in the ring in order that it might be possible to determine at any time the residual flux in the core. This determination was made by means of the galvanometer and test coil previously calibrated. With the coil encircling the ring on the side opposite the gap, the plug was removed and the throw of the galvanometer noted. The coil was then quickly slipped off the ring and a second deflection observed. Since the calibration showed 32.62 lines of induction per mm. deflection, the total flux was found by multiplying 32.62 by the sum of the two deflections.

A series of photographs was taken for the glass-hard steel with maximum demagnetizing field ranging from 1.65 gauss to 34.5 gauss. The variation of induction in the first case was 114 lines of induction per sq. cm. while in the latter it was 1,960. In this latter case the remanence was 140 lines per sq. cm. and the application of the current caused a reversal of the magnetism, producing a negative induction of 1,820 lines per sq. cm. As the current was reduced to zero the magnetic flux returned to its former positive value. With somewhat smaller demagne-

tizing currents the magnetism was not reversed and the recovery on reducing the field to zero was from a value of the induction which was still positive to a higher positive value. For example, when the current was 1.56 amperes, giving a field of 27.4 gauss, the induction fell from 1630 to 60 and returned to 1,630 when the current became zero. The magnitude of these recoveries is appreciated when they are compared with the maximum residual magnetism observed in the specimen. This latter value was 8,036 lines per sq. cm.; the recovery of 1,960 is therefore 24.5 per cent. of the maximum residual induction.

The curves for this specimen were taken in order beginning with small demagnetizing current. The first application of the current produced a certain amount of permanent demagnetization. After many applications of the current the magnetism reached a stable condition in which the recovery was just equal to the demagnetization. Then the photograph was taken. Next a larger current was applied which produced a further permanent demagnetization and a new condition of stability. Then another photograph was taken. The method of determining the residual flux after each photograph was taken has been described. The removal of the plug before each observation introduced a demagnetizing field due to the ends and caused a marked decrease of the residual magnetism when the flux was at its high values. This effect decreases as lower values of the remanence are reached and vanishes when the flux becomes zero. These curves were plotted with magnetic flux as ordinates rather than the flux density or induction while the figures quoted above are values of flux density. The area of the specimen was 1.4 sq. cm. hence to obtain variation of induction from the curves the ordinates should be divided by 1.4.

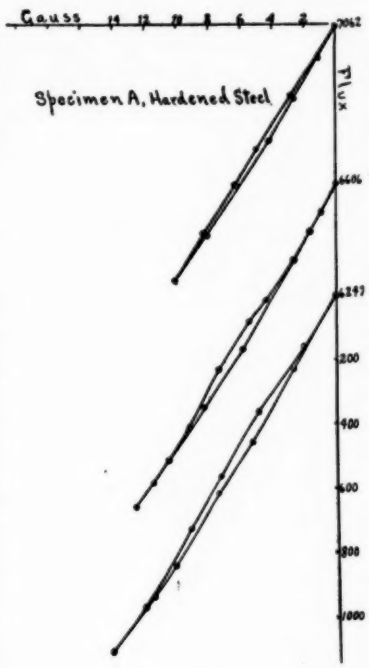


Fig. 3.

Figs. 2 and 3 are two of the series obtained from this specimen. The curves were all transferred to a single sheet (see Fig. 4) and properly placed with reference to the axes according to the residual flux determined after each was taken.

Data for Demag. and Recovery Curves of Specimen A.

Current.	No. of Secondary Turns.	Def. on Removing Plug.	Def. Due to Remaining Flux.	Remanent Flux.	Maximum *H*.	Gauss per Division.
.094	125	24.0	207	7,535	1.6537	.0438
.48	75	21.0	198	7,144	8.4446	.1206
.576	50	20.5	196	7,062	10.1336	.1842
.72	50	19.5	183	6,606	12.667	.2112
.80	50	18.5	173	6,247	14.074	.2346
.92	50	17.5	159	5,757	16.185	.234
1.00	25	16.0	146	5,284	17.593	.3824
1.18	25	14.0	122	4,436	20.76	.3844
1.32	25	12.0	99	3,621	23.222	.3776
1.40	25	10.0	88	3,197	24.63	.4561
1.56	25	7.0	63	2,283	27.445	.524
1.68	25	6.0	49	1,794	29.556	.5185
1.80	25	3.5	32	1,158	31.667	.5278
1.96	25	0	6	197	34.482	.6000

Data for Demagnetization and Recovery Curves Derived from the Current Curve and the Flux Variation Curve.

H = field intensity. ϕ = magnetic flux.

H	ϕ	H	ϕ	H	ϕ	H	ϕ
.24	22.3	.75	70	1.105	107	1.02	95
.48	49.9	2.08	168	2.763	233	2.95	238
.85	81.8	3.98	308	4.973	399	4.50	368
1.40	136.0	5.18	401	6.263	513	5.60	440
1.65	159.0	6.51	522	8.289	668	7.50	584
1.36	133.0	8.44	656	10.133	812	11.55	938
.70	68.0	6.93	559	8.289	668	12.67	1015
.48	46.2	5.43	437	6.080	501	10.60	862
		3.74	308	4.140	370	7.50	582
		2.65	219	2.763	239	5.58	438
		1.03	93			4.50	365
						1.89	155
2.111	167	1.872	191	1.912	160	1.53	130
4.93	370	4.446	406	4.589	389	3.84	321
7.27	573	7.956	668	8.413	664	7.88	641
9.15	740	10.070	859	10.707	870	11.92	916
11.96	979	13.104	1134	14.149	1164	14.61	1214
14.07	1110	16.180	1432	16.443	1374	18.45	1540
12.43	1014	13.104	1183	17.59	1466	20.76	1740
10.09	847	9.600	919	16.44	1374	18.07	1557
7.51	621	6.790	692	14.53	1213	15.00	1305
5.40	465	4.446	441	11.472	962	11.15	985
2.81	239	14.04	167	7.839	733	7.88	733
				5.162	504	4.23	378
				1.912	183	.87	114

H	ϕ	H	ϕ	H	ϕ
3.77	321	1.82	137	1.57	160
8.31	664	4.11	343	4.19	400
12.46	1054	8.66	710	9.43	836
15.86	1374	12.77	1053	14.67	1150
20.01	1740	16.41	1397	18.34	1534
23.26	1992	20.75	1786	23.31	1946
20.39	1786	24.86	2038	27.45	2198
16.61	1511	21.89	1866	23.80	1969
12.08	1145	18.24	1580	19.38	1649
8.68	847	13.68	1191	18.34	1237
4.91	458	9.12	893	10.18	962
		5.93	470	5.67	481
		1.82	149	1.05	137
1.80	183	2.64	251	4.5	378
4.67	492	7.38	618	9.9	801
10.37	939	13.20	1100	16.8	1340
16.07	1340	18.50	1511	22.8	1763
20.48	1800	23.75	2015	27.3	2255
25.93	2233	29.46	2427	34.5	2748
29.55	2496	31.67	2611	27.3	2358
25.93	2233	30.08	2473	22.2	1981
21.26	1855	26.64	2267	15.3	1488
15.54	1397	21.00	1855	9.9	1008
10.89	1031	14.98	1397	4.5	423
5.70	527	10.55	973		
1.39	137	4.22	424		

It will be observed that the extreme points of the demagnetization curves lie on a smooth curve. The determination of the relation of this curve to the maximum hysteresis curve was next undertaken. This portion of the hysteresis curve was determined ballistically and was found to lie outside the locus of the extreme points and as would be expected deviated widely from it near the upper end. This deviation was due to the end effects established when the plug was removed as previously explained. A second ballistic curve was traced under the same conditions that existed when the work on the oscillographic curves began, *i. e.*, the core was highly magnetized, then the plug was removed and replaced, thus the remanence was brought down to the same value at which it stood when the oscillographic work began. This second ballistic curve follows quite closely the contour of the locus of the extreme point of the demagnetization curve as is seen on Fig. 4. However, it lies outside of this locus as does the maximum hysteresis curve.

That the relation of these two curves is not due to error is shown by further work by the ballistic method. The specimen was first highly

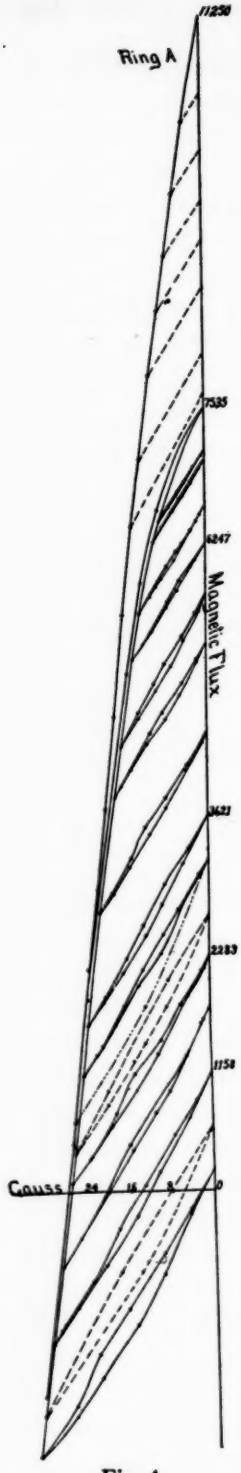


Fig. 4.

magnetized, then a certain demagnetizing current applied sufficient to bring the residual magnetism down to the region in which the oscillographic curves were obtained. Conditions would then be represented by some point on the maximum hysteresis curve. From this point the curve of recovery was traced ballistically, as many points being determined as seemed expedient. This curve was found to differ from that obtained by the oscillograph for the same demagnetizing current both in position and slope. The conditions differed in the two cases only in this: the ballistic curve was obtained the first time the steel was put through the cycle while the oscillographic curve was taken after several thousand repetitions of the cycle. It was next found that after opening and closing the circuit through the primary, a second ballistic curve approached more nearly the oscillographic curve. When the key in the primary circuit was opened and closed about a hundred times and the ballistic curve then traced, it was found that its extreme point fell almost exactly upon the locus of the extreme points of the oscillographic curve, and furthermore the slope and shape of the new curve agreed with those of the oscillographic curve. A number of cycles of demagnetization and recovery were traced ballistically. Some of them are shown on Fig. 4. in their relation to the other curves.

In plotting the hysteresis curve by the ballistic method, an interesting fact appeared regarding end effects. The residual induction after the application of a strong magnetizing field and before the plug was taken out was about 8,000 lines of induction per sq. cm. When the plug was removed the end effects caused it to fall to a value of 4,940 lines per sq. cm. The core was then remagnetized with the plug in place and a demagnetizing current applied. This current might be of any strength provided it did not reduce the induction below 4,940. The circuit carrying this current was then opened and a certain recovery ensued. Then the plug was removed and it was found that the residual induction was again 4,940. This constancy of the residual

magnetism on open magnetic circuit persisted until a demagnetizing filed of 12.8 gauss had been applied and then suddenly the residual magnetism, with the magnetic circuit open, fell to lower values. The point at which it breaks down is the point where the recovery on removing this demagnetizing field of 12.8 gauss is equal to the demagnetization due to the ends when the plug is removed. Fig. 5 shows these relations in graphic form. Curve *A* represents the maximum hysteresis curve. The inercept between *A* and *B* is the recovery upon breaking the circuit. The intercept between *B* and *C* is the demagnetization following the removal of the plug. At the right of point *P* the demagnetization due to the ends exceeds the recovery while at the left of *P*, the recovery is

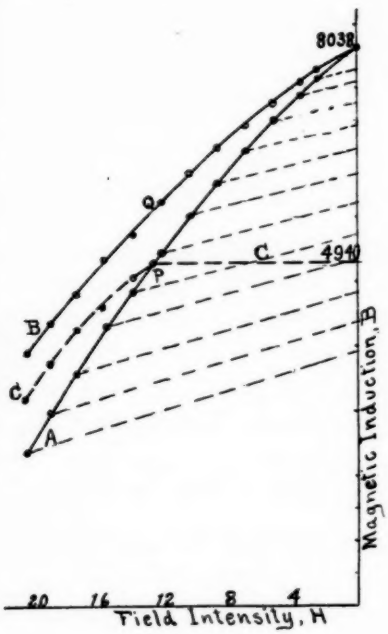


Fig. 5.

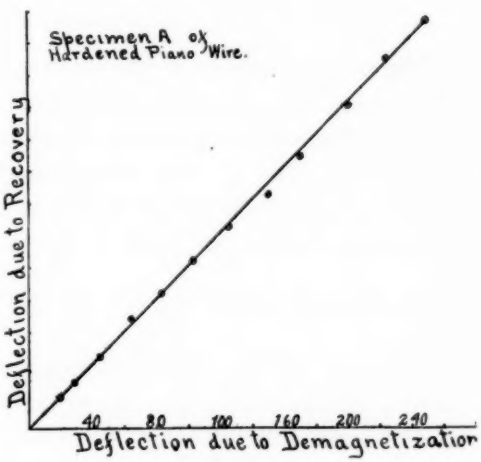


Fig. 6.

greater. Intercept *PQ* represents both demagnetization due to the ends and recovery after removal of the field and it is at this point that the residual magnetism with the gap open ceases to be constant. The interpretation of this horizontal portion of curve *C* seems to be this: that there is a welldefined upper limit beyond which the remanence of a steel core with exposed ends cannot be made to pass.

As has been mentioned, the recovery curves for a given specimen are practically parallel. For very hard steel the portion of the hysteresis curve with which we are concerned does not differ much from a straight

line. If it were exactly a straight line, it would follow that the recovery would be strictly proportional to the difference between the maximum residual flux and the residual flux corresponding to the recovery curve. The line of Fig. 6 is presented to show how nearly this statement of proportionality approximates the truth.

In designing magnetic circuits, so much attention is usually given to reducing the magnetic reluctance of the joints that it seems desirable to find out if the reluctance of the joints had any influence upon the preceding experiments. A demagnetization and recovery curve was obtained by the oscillograph with the plug out of the circuit; thus the magnetic circuit contained an air gap about one centimeter long. The only marked difference between this curve and the others was a decrease in slope amounting to about thirteen per cent. This would lead to the conclusion that when the gap was filled with soft iron, the effect of the joints, upon the form or slope of the demagnetization and recovery curves would be insignificant.

V. Experiments with Specimen B.

The second specimen studied was a ring of no. 4 piano wire in its normal condition, *i. e.*, without heat treatment. It consisted of 1,215 wires each having a cross-sectional area of .0012 sq. cm., the aggregate area being 1.458 sq. cm. The mean length of the ring was 32.67 cm. The secondary coil consisted of five sections of 25 turns each. Outside the secondary, the primary coil of 350 turns was wound. The intensity of magnetic field within the metal was $\frac{4\pi \cdot 350\, i}{10 \cdot 32.67}$ gauss, i being the current in amperes. When a test coil of fourteen turns was used, one mm. deflection of the ballistic galvanometer indicated a change of flux of 32.62 lines or a change of flux density (B) of $32.62/1.458 = 22.37$. In dealing with softer specimens, which are much more susceptible to end effects, it seemed best to try to do the work without making any air gap in the magnetic circuit, hence this ring and two others of soft iron which were used were not cut.

Plate 7 shows the hysteresis curve by the oscillographic method when the maximum field intensity is about 62 gauss. The residual induction is seen to be 13,665. As a check upon this curve a determination of the residual inductance was made by the ballistic method using the same maximum magnetizing field and it was found to be 13,670. Such close agreement is probably the result of a fortunate observation as it is beyond the limits of accuracy that could reasonably be expected.

A series of demagnetization and recovery curves was obtained for this specimen by the oscillographic method. They differ in slope from those of the former series but otherwise show very little difference.

Since there was no opening in the magnetic circuit, the method used in the former case of determining the position of the curves with reference to each other and to the residual induction could not be employed. In the case of ring *A* the curves were placed each with its highest point at the point representing the residual flux actually measured after the curve was photographed and the curve through their extreme lower points was drawn. The results of that work would warrant us in taking the reverse process this time. The locus of the extreme points lies within the maximum hysteresis curve. A hysteresis curve that approaches the maximum hysteresis curve very closely can be obtained by the ballistic method but just where the locus of the extreme point lies with reference to this curve is a matter to be determined. This was done by means of a Koepsel permeameter. A field of 134 gauss was applied to a specimen of this wire, this value of field intensity being chosen in order to reproduce the conditions under which the ballistic curve was taken. The field was then reduced to zero leaving a remanence of 15,400 lines per sq. cm. A small demagnetizing field was next applied and the value of the flux density, *B*, observed from the instrument. Then the circuit through the magnetizing coil was opened and closed 50 times and another reading of *B* taken. The key was then opened and closed 50 times more and the value of *B* again observed. Next a larger demagnetizing field was applied and the process repeated. Fourteen points on this portion of the hysteresis curve were thus found and also fourteen on a curve lying within the hysteresis curve. This latter curve, it was assumed would represent the locus of the extreme points of the recovery curves provided the hysteresis curve were accurate. In using the permeameter it is necessary to correct the observeed curve by adding to its abscissæ the abscissæ of a shearing curve. No shearing curve

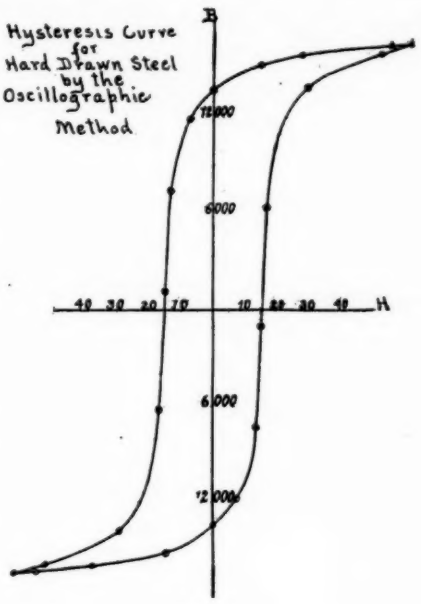


Fig. 7.

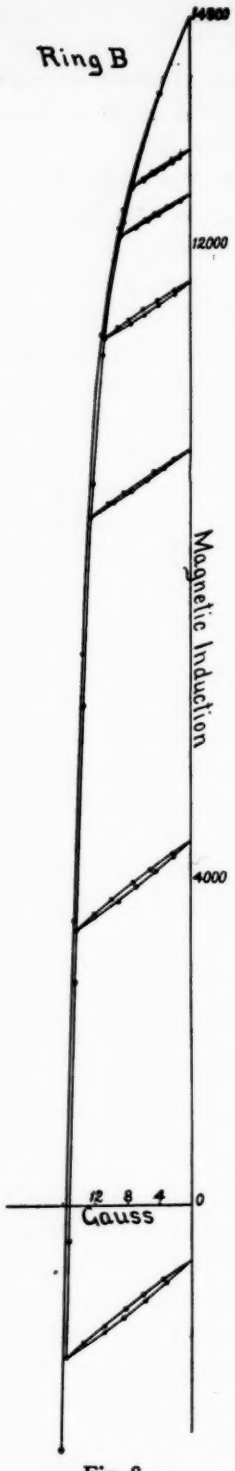


Fig. 8.

was at hand that could be assumed to be accurate for this specimen and so the ballistic curve shown in Fig. 8 was used. The curve of the extreme points of the recovery curves would lie within this hysteresis curve and it was assumed that the difference in ordinates of these two curves would be the same as the difference between the ordinates of the corresponding curves given by the permeameter. In this way the locus of the extreme points was determined. Both these curves are shown in Fig. 8. The oscillographic curves of demagnetization and recovery were now reduced to the same scale and placed upon the sheet in such a way that their extreme points fell upon the curve which lies within the hysteresis curve. The maximum recovery observed was 1,200 lines per sq. cm. and occurred in the region of zero residual flux while the maximum remanence was 14,900, thus the recovery is about 8 per cent. of the maximum remanence. The maximum recovery for the hardest steel was 24.5 per cent. of the maximum remanence. This large difference is due both to a smaller recovery and to a larger remanence in the second case.

Attention has been drawn to a number of analogies between elastic and magnetic phenomena. Bridgman[1] has recently done a piece of work on "The Collapse of Thick Cylinders under High Hydrostatic Pressure" that has furnished curves bearing a striking resemblance to the curves of Figs. 4 and 8. Fig. 13 is a copy of one of Bridgman's figures inverted for the purpose of comparison with the magnetic curves. The specimen is a hollow steel cylinder and the strain is measured by the change of internal volume. Three points of resemblance are to be observed: (1) The hysteresis, (2) the increase of elastic limit with overstrain, (3) the effect of repeated application of the same stress as shown near the bottom of the figure. The actual change of elastic limit with over-strain is of course much less than the apparent change, a large part of that appearing in the figure being due to change of dimensions of the specimen.

[1] PHYS. REV., January, 1912, p. 1.

The demagnetization and recovery discussed in the previous pages have taken place under conditions such that the metal was carried during each cycle to the limit of its elasticity for the particular remanence to which it returned. For lower values of remanence the elastic limit is higher. The question would naturally arise as to the effect of a demagnetizing field that did not approach the elastic limit. The shape of the curves already taken would lead one to expect to find the change of flux due to a given demagnetizing field very nearly independent of the residual flux. This would follow from the fact that magnetic stress and strain have been found approximately proportional. Lord Rayleigh[1] has shown that the change of induction resulting from a very small change of field intensity is independent of the magnetic state of the iron, or to quote his conclusion, "That the value of the susceptibility to small changes of force is approximately independent of the initial conditions as regards force and magnetization until the region of saturation is approached." He was dealing with field strengths ranging from .04 gauss to .000018 gauss. The oscillographic method was applied to this problem, using a much larger demagnetizing field. The results may be made clear by referring to two curves produced by the same field intensity. In one case the residual flux density was 12650 and in the other 500 lines of induction per sq. cm. The maximum field intensity was 7.27 gauss. The reduction in flux density from the higher remanence was 440 while it was 475 from the lower. This represents a difference of about 8 per cent., and means that the susceptibility to small changes of field intensity increases as the remanence decreases. Similar results were obtained by the ballistic method.

Data for Demagnetization and Recovery Curves of Specimen B.

$$\text{Max. } H = \frac{4\pi \cdot 350 \cdot i}{10 \cdot 32.67} = 13.46\, i.$$

i being the current.

Current.	No. Turns in Secondary.	Maximum H.	Gauss per Division.
.27	1.25	3.6342	.0512
.54	.50	7.2684	.1191
.54	.50	7.2684	.1483
.64	.75	8.616	.1325
.79	.50	10.6334	.1715
.92	.50	12.385	.1567
1.06	.50	14.2676	.1719
1.17	.25	15.75	.2045
1.13	.50	15.21	.1925

[1] Phil. Mag., 23, 1887, p. 225.

Data for Demagnetization and Recovery Curves of Specimen B.

Value of one division in gausses taken from preceding table. Value of one division in lines of magnetic induction determined by the calibration. H = field intensity. B = variation of induction.

H	B	H	B	H	B
.427	25	.476	23	.89	41
1.254	77	1.071	82	2.14	139
2.330	160	2.50	156	3.40	213
3.020	212	3.69	225	4.59	286
3.630	265	4.78	303	5.77	368
2.690	219	5.97	368	7.27	475
1.970	164	7.27	440	5.77	384
.947	77	5.35	327	4.37	303
.358	32	3.94	254	3.18	238
		2.91	180	2.07	156
		1.67	98	1.03	74
		.70	35		
1.06	58	1.95	110	1.88	122
2.78	150	4.13	240	4.39	262
4.19	228	5.90	344	6.11	393
5.51	322	7.70	483	8.15	540
7.50	439	9.06	573	10.18	667
8.62	511	10.63	704	12.38	835
6.62	408	7.35	532	9.40	647
5.24	319	5.64	426	7.13	516
3.84	233	3.93	311	5.33	377
2.32	139	1.79	172	3.13	221
.99	60			1.25	82
2.15	156	1.02	71	1.16	49
4.99	328	3.58	235	1.93	98
7.23	491	6.14	416	5.77	266
9.92	695	8.28	585	8.08	385
12.04	880	11.04	806	9.72	491
14.27	1064	13.50	1015	12.80	638
12.04	901	15.75	1194	15.21	761
8.94	737	10.84	871	12.03	610
6.88	573	8.59	699	8.18	454
4.30	360	5.83	487	6.45	361
2.06	180	3.07	267	1.35	74

VI. Experiments with Specimen C.

Specimen C is a ring of transformer iron of mean length 37.93 cm. and cross-sectional area 2.07 sq. cm. The primary coil has 325 turns, hence the magnetizing field is

$$\frac{4\pi \cdot 325\, i}{10 \cdot 37.93} = 10.767\, i.$$

One division deflection of the galvanometer corresponds to a change of induction of 15.76 lines per sq. cm. when there are fourteen turns in the test coil.

When the demagnetization and recovery curves for this specimen were plotted it was apparent that they were somewhat different from those obtained from the steel specimens. The hysteresis seemed much larger. A part of this was due to the scale to which the curves were plotted, but when allowance was made for that, they still seemed to have an abnormally large area. This peculiarity of the curves for this specimen is undoubtedly due to eddy currents in the iron. These currents are in opposition to the time rate of change of magnetic flux and hence lag behind the magnetizing current by an amount somewhat in excess of 90°. They produce a magnetic field one component of which is in direct opposition to the applied field while the other reacts to cause a lag in phase of the field behind the current. Since the curves are plotted on the assumption that the applied field is proportional to the ordinate of the current curve, it is evident that the lag of induction behind the field intensity as given by the curves would be greater than the true hysteretic lag. When there is hysteresis without eddy currents the induction shows a lag behind the current at some points along the curve but none whatever at points of maximum or minimum field intensity. A difference of phase between the two curves on the photographs at their highest points is therefore evidence of the disturbing influence of eddy currents. Furthermore, if a method of correcting for eddy currents can be found, the phase relation of the highest points will furnish the criterion for determining the magnitude of the correction. Eddy currents are produced by the variation of the flux, ϕ, and their magnitude could be expressed as $K' (d\phi/dt)$, where K' is a negative constant. The current in the secondary, which operates one loop of the oscillograph, is also proportional to the derivative of the flux with reference to the time and the curves are marked $K (d\phi/dt)$ on the figures. Some fraction of $K (d\phi/dt)$ will equal $K' (d\phi/dt)$, in other words some fraction of the ordinate of the secondary curve on the oscillographic record will constitute the correction that should be applied to the corresponding ordinate of the current curve to give the ordinate that is proportional to the actual field intensity. It is comparatively easy to tell by inspection of the curves what this fraction is; or at least it may be found after two or three trials in applying the correction to the current curve. The flux and its time rate of variation are behind the current in phase at the highest points of the curves. If the fraction chosen be too small, the flux will be behind the corrected curve while if the fraction chosen be too large the flux curve will

fall ahead of the new curve in phase at the highest points. A value of the fraction is chosen which places the corrected curve in phase at the highest point with the flux curve and then the ordinates of this new curve represent the resultant or actual field intensity applied to the iron. Fig. 9 shows the corrected curve with points indicated by the small circles. The correction applied to each ordinate of the curve was 13 per cent. of the corresponding ordinate of the secondary curve. The hysteresis for this specimen, as shown by the corrected curves, is found to differ very little from that shown by the other specimens for an equal change of flux.

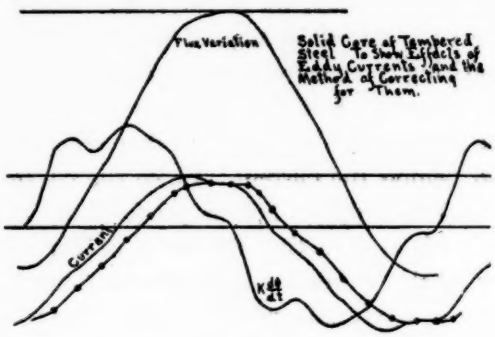

Flux Variation
Solid Core of Tempered Steel To Show Effects of Eddy Currents and the Method of Correcting for Them.
Current
K dB/dt

Fig. 9.

In order to further test this method of correcting for eddy currents, a solid magnet of tempered steel was used. Eddy currents would certainly be large in this case, but it was arranged to reduce end effects to a negligible magnitude by filling the small air gap with a carefully fitted block of iron. Fig. 10 shows the results. The curve plotted from the corrected data shows no more hysteresis than was found for the wire ring.

To obtain further data on the relative magnitude of the hysteresis in steel and soft iron for the kind of cycle here considered, a ring of soft iron wire was used. Wire having a diameter of .96 mm. was chosen. The wire of specimen *A* had a diameter 1.03 mm. If size of wire were all that determined the eddy currents effect, we should be assured of less disturbance from them with this new ring than was experienced with specimen *A*. However, in getting the total hysteresis curve this is not the case. With soft iron the hysteresis curve is nearly parallel to the axis of ordinates for a long way and hence the time rate of change of flux along this part of the curve is much greater than it is for hard steel where the slope of the curve is much less, hence in plotting the total hysteresis curve of soft iron, we should expect greater errors due to eddy currents than would arise in the case of hard steel. This argument, however, does not apply to demagnetization and recovery curves since

the slope does not vary greatly from point to point along the curve and the time required for demagnetization or recovery is 1/120 of a second irrespective of the magnitude of the change. Hence for a given flux variation and for a given size of wire, effects due to eddy currents for soft iron ought not to differ widely from those in steel. Curve *D* of Fig. 12 was obtained from this new ring of soft iron wire. The phase difference at the maximum and minimum points of the current and flux curves was so slight that no attempt was made to correct for eddy currents. Apparently the hysteresis is somewhat larger than for the steel

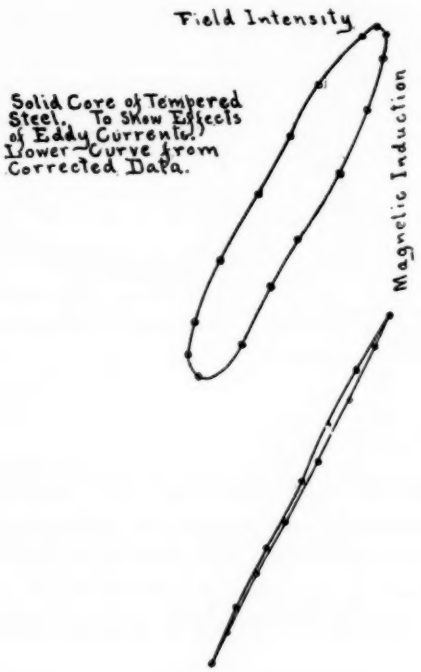

Field Intensity
Solid Core of Tempered Steel. To Show Effects of Eddy Currents. Lower Curve from Corrected Data.
Magnetic Induction

Fig. 10.

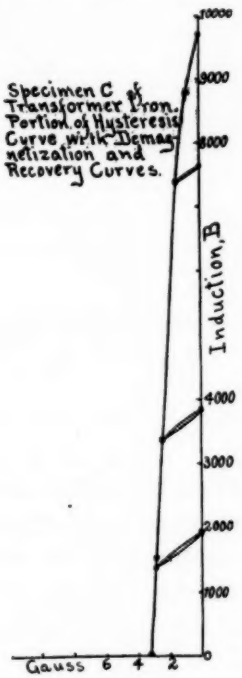


Fig. 11.

specimens. The conclusion is certainly justified that for cycles of the kind here considered the hysteresis of very hard steel is not greater than that of soft iron for an equal variation of flux density and it may be somewhat less.

The hysteresis curve for ring *C* was plotted ballistically in Fig. 11 and the recovery curves drawn to the same scale as the hysteresis curve are also shown in their relation to the residual magnetic induction. The recovery in the region of small residual magnetism in this case is only about 6 per cent. of the maximum residual induction.

Fig. 12 is for the purpose of comparing the slopes of the curves for the different specimens.

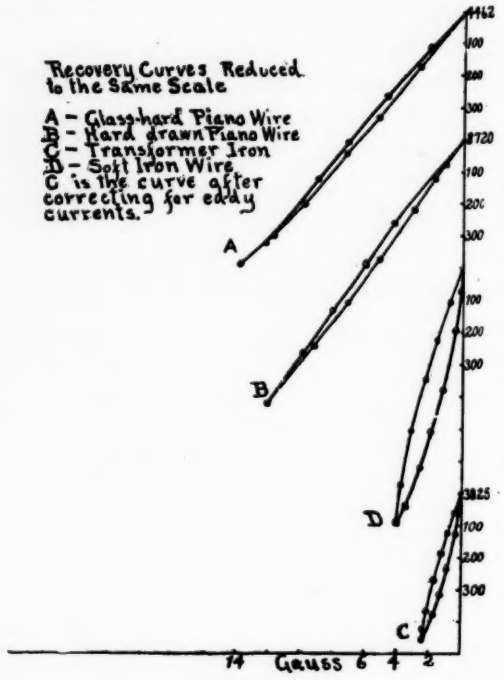

Recovery Curves Reduced to the Same Scale
A – Glass-hard Piano Wire
B – Hard drawn Piano Wire
C – Transformer Iron
D – Soft Iron Wire
C is the curve after correcting for eddy currents.
A
B
D
C
14 Gauss 6 4 2

Fig. 12.

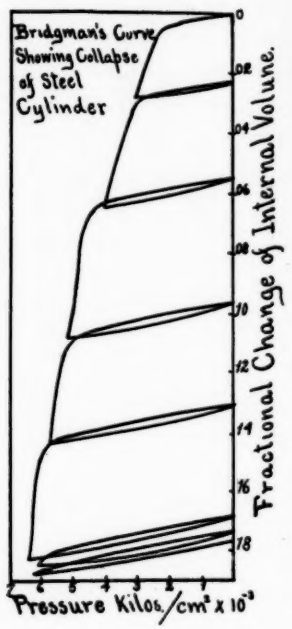


Fig. 13.

C. Conclusions.

1. An oscillographic method has been developed for plotting any kind of a magnetic cycle with an alternating current of 60 cycles provided eddy currents and end effects are negligible. The method is applicable when the flux variation is as small as 150 lines of induction and it may be used for any cycle in which the flux variation is greater than that amount.

2. It was shown that in a given specimen, for any value of residual magnetism, there is a definite elastic limit which is the maximum field that can be applied to the specimen without producing permanent demagnetization; and that if any field not exceeding the limiting field be applied and then removed, the specimen will return to its initial condition.

3. Within the elastic limit, magnetic stress and strain are approximately proportional, *i. e.*, the demagnetization is approximately proportional to the field intensity that produces it, provided the specimen returns to its original remanence when the field is reduced to zero.

4. The hysteresis as indicated by the area of the curves is small and is approximately the same for soft iron and hard steel.

5. The extreme points of the stable demagnetization and recovery curves lie inside the maximum hysteresis curve.

6. The susceptibility to small demagnetizing fields increases slightly with decrease of residual magnetism, the maximum difference observed being about eight per cent.

7. The recovery from the effect of a demagnetizing field is greater for low value of residual induction. For hardened steel it was found to be 1,960 lines of induction per sq. cm. or 24.5 per cent. of the maximum residual induction. For drawn piano wire it was 1,200 lines per sq. cm. or 8 per cent. of the maximum remanence and for soft iron it was 600 lines per sq. cm. or 6 per cent. of the maximum remanence. These numbers represent the elastic limits for low values of residual magnetism.

8. There is shown to be a well-defined upper limit beyond which the residual magnetism of an open magnetic circuit of steel can not be made to pass.

9. The slope of the demagnetization curves for hard steel is 57. For softer specimens it is larger and it reaches a value for very soft iron of about 180.

10. For hard steel the elastic limit is approximately proportional to the difference between the residual magnetism and the maximum residual magnetism.

The work was done in the physical laboratory of the University of Michigan under the direction of Professor K. E. Guthe. The author wishes to express his appreciation of Professor Guthe's interest in the work and his helpful suggestions while it was in progress.

UNIVERSITY OF MICHIGAN,
ANN ARBOR, June 1, 1912.

NOTE ON THE VELOCITY OF ELECTRONS LIBERATED BY PHOTOELECTRIC ACTION.

BY KARL T. COMPTON.

THE relation between the velocities of the electrons liberated by the action of light on metals and the wave-length of the incident light has been widely studied and with quite conflicting results. The superposition on the effect under investigation of secondary effects due to reflected light and reflected electrons, to variable contact difference of potential between the emitting and receiving electrodes, to electrostatic leakage, to impurity of the spectrum and to several other factors has proved of particular annoyance in this work. In recent work by Prof. O. W. Richardson and the writer[1] these sources of error seem to have been largely eliminated and quite consistent results were obtained supporting the view that the velocity of the liberated electrons is a linear function of the frequency of the incident light. By a quite independent method and at about the same time A. L. Hughes[2] reached similar conclusions. These results agree that the relation between the velocity and the frequency may be accurately expressed by an equation of the form

$$V = kn - w_0,$$

where V is the initial kinetic energy of the liberated electrons, expressed in equivalent volts, k and w_0 are constants and n is the frequency of the incident light. This is the type of equation which was first developed by Einstein[3] on the unitary theory of light and recently by O. W. Richardson[4] by thermodynamic and statistical methods which do not necessarily involve the unitary light theory. In it w_0 is the work, expressed in equivalent volts, which is done in carrying an electron from the inside to the outside of the metal and k equals $300h/e$, where h is Planck's constant and e is the charge on an electron. On these theories therefore

$$V = \frac{300h}{e} n - w_0.$$

[1] Phil. Mag., Vol. 24, p. 576 (1912).

[2] Phil. Trans. of the Roy. Soc. of London, Series *A*, Vol. 212, p. 205 (1912).

[3] Ann. der Phys., Vol. 17, p. 146 (1905).

[4] PHYS. REV., Vol. 34, p. 146 (1912); Phil. Mag., Vol. 23, p. 615 (1912); ibid., Vol. 24, p. 570 (1912); Science, Vol. 36, p. 57 (1912).

The experimental results have accurately confirmed the linear relation between V and n, although they have led to values of h slightly smaller than the generally accepted value 6.55 $(10)^{-27}$ erg sec. The average value of k, for eight different metals, was found by Professor Richardson and the writer to lead to $h = 5.4\ (10)^{-27}$ erg sec., while Hughes' results for a number of metals lead to $h = 5.6\ (10)^{-27}$ erg sec.

A different formula has been derived by Jakob Kunz[1] on the theory that light consists of discrete vibrating Faraday tubes. He arrives at the conclusion that the energy of the liberated electrons varies as the square of the frequency and that the relation may be expressed by

$$V = sn^2 - w_0$$

where w_0 represents the work done by an electron in escaping from the metal and s is a constant.

In a recent paper Mr. David W. Cornelius[2] published results which claim to support Kunz's theory and to discredit the Einstein-Richardson formula. In this paper there are certain errors and inconsistencies which if corrected make the support of Kunz's theory much less convincing and which in fact seem to lend support rather to the theories of Einstein and Richardson. In view of the importance of settling this matter it has seemed to the writer worth while to briefly discuss the results of Mr. Cornelius and to compare the two types of theories in the light of evidence from direct experiment and from associated phenomena.

Mr. Cornelius experimented with potassium and cæsium in glass bulbs, prepared by melting or distilling *in vacuo*. He illuminated the metal by light from a carbon arc. The light was rendered parallel by a system of lenses and was then passed through a prism which directed the desired beam of light on to a slit in front of the photoelectric cell. Different wave-lengths of light were directed on the slit by rotating the prism.

With this arrangement Mr. Cornelius measured the maximum potential acquired by the receiving electrode when various wave-lengths of light were directed on the metal. He plotted these values of the potential against the corresponding wave-lengths and obtained a curve which is concave upwards. He then drew this conclusion: "The curve certainly is not a straight line as Planck's law would require." The fact is, however, that Planck's law (or Einstein's formula) requires a linear relation between the potential and the frequency and not between the potential and the wave-length. This last relation is represented by a hyperbola, as is shown in Fig. 1, and it is this curve with which Mr. Cornelius' curve

[1] PHYS. REV., Vol. 29, p. 3 (1909); ibid., Vol. 30, p. 212 (1909).
[2] PHYS. REV., Vol. 1, series 2, p. 16 (1913).

should have been compared. The only justification for calling the curve a straight line would be in case only a small section of the spectrum were considered. Mr. Cornelius worked between wave-lengths 420 $\mu\mu$ and 750 $\mu\mu$, which represents the section *ab* of the curve in Fig. 1 and which has a distinct curvature. This test of the Einstein-Richardson formula is therefore misleading.

The correct test of the two theories is given by plotting volts against frequency in the one case and against the square of the frequency in the other. I have done this for a typical set of Mr. Cornelius' readings for potassium in Fig. 2. Curve 1 represents the variation of the initial energy of the electrons with the frequency, Curve 2 represents this variation with the square of the frequency and Curve 3 represents the variation with the cube of the frequency. The units along the abscissa are 10^{15}, 10^{30} and 10^{45} respectively. According to Einstein's or Richardson's theories Curve 1 should be a straight line. According to Kunz's theory Curve 2 should be a straight line. As a matter of fact neither is a straight line and the experimental results fit most closely a variation with the cube of the frequency. It is true that the linear relation is rather more closely followed in Curve 2 than in Curve 1. However, this slight support of Kunz's theory is more than offset by the following evidence.

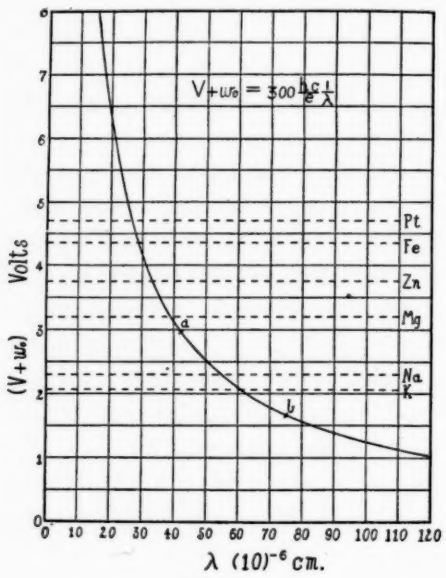


Fig. 1.

Assuming Kunz's theory to be correct, the slope of Curve 2 should give the constant s; and the point at which the curve, extended, cuts the axis of volts should give the constant w_0, or the work expressed in volts done by an electron in escaping from the metal. The position of intersection is rather uncertain owing to the curvature of the curve. But taking the most favorable possible case and drawing the straight line *ab* we find that it cuts the axis of volts at a point making $w_0 = 0.35$ volts. This is the largest value for the work done by an electron in escaping which we can obtain from the readings and it is about the same value which Kunz[1] calculates for several other metals according to his formula.

[1] PHYS. REV., Vol. 31, p. 536 (1910).

But this value is impossibly small, being less than the average energy of thermal agitation at 23° C., which is equivalent to 0.38 volts. If it were correct it would require that in the course of their thermal agitation more than half of the electrons striking the surface of the metal perpendicularly during any interval of time would escape. Kunz's theory therefore leads to values of w_0 which are impossible and not even of the right order of magnitude.

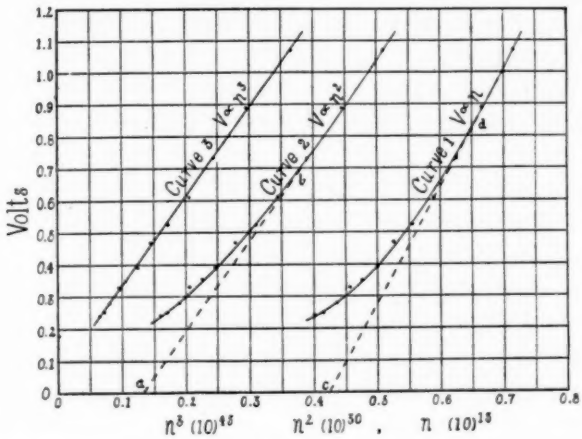


Fig. 2.

Consider now Curve 1 in relation to the Einstein-Richardson formula. Take the line *cd* to represent most favorably the linear relation between V and n. It will be shown later that the points on the lower part of the curve are less trustworthy than those above. The slope of this line is the constant k from which we should be able to derive Planck's constant $h = 6.55\ (10)^{-27}$ erg. sec. The actual slope leads to a value $h = 5.7\ (10)^{-27}$ erg sec., which is not a serious disagreement with the theory and which is very close to the values previously mentioned in connection with the results of O. W. Richardson and the writer and of A. L. Hughes. And the intersection with the axis of volts gives $w_0 = 1.6$ volts, which is not far from the value of the work done by an electron in escaping from potassium as indicated by other methods.

As a matter of fact measurements of w_0 have been made by three independent methods for a number of metals including platinum and the similar metal osmium.[1] Probably osmium gave the most reliable value

[1] O. W. Richardson, Phil. Trans., *A*, Vol. 201, p. 497 (1903); Vol. 207, p. 23 (1906); H. A. Wilson, ibid., Vol. 202, p. 243 (1903); F. Deininger, Ann. der Physik, Vol. 25, p. 296 (1908); O. W. Richardson and H. L. Cooke, Phil. Mag., Vol. 20, p. 173 (1901); H. L. Cooke and O. W. Richardson, PHYS. REV., series 2, Vol. 1, p. 71 (1913).

as about 4.7 volts, and w_0 for platinum is certainly very near to this value. A calculation of w_0 for platinum from photoelectric measurements by a process similar to that followed in the preceding paragraph gave $w_0 = 4.4$ volts.[1] Thus for platinum, the only metal for which we have very reliable measurements of w_0 to use in testing the theories, the Einstein-Richardson formula is verified.

It is possible however to estimate w_0 for potassium and other metals indirectly from the contact difference of potential between platinum and the metal considered, for O. W. Richardson[2] has shown that for two metals s and p

$$w_s - w_p = e\left\{ V_p - V_s - \theta \frac{\partial}{\partial \theta}\left(V_s - V_p \right) \right\},$$

where $V_p - V_s$ is the contact difference of potential between the metals and e is the charge on an electron. The last term which represents the Peltier effect may be neglected and therefore, knowing $V_p - V_s$ and w_p, the work in ergs necessary to withdraw an electron from platinum, we are able to calculate w_s for the metal considered. In fact the difference between the works required to extract an electron from each of two metals, expressed in equivalent volts, is simply equal to the contact difference of potential between the metals. There is abundant qualitative support for this and it is in harmony with the facts of thermionic emission. If we determine w_0 for potassium in this way we find that $w_0 = 2.05$ volts, which is not far from the value found from Curve 1.[3]

Later in his paper Mr. Cornelius calculates theoretically the work done in the escape of an electron moving to infinity against the electrostatic attraction of the parent atom, from a distance equal to the radius of the atom. It has long been known that such a calculation gives values for w_0 of the right order of magnitude. The amount of work which must be supplied by the light to expel an electron from potassium is thus calculated by Mr. Cornelius to be $4.57\ (10)^{-12}$ ergs. The interesting fact is that if this value is reduced to equivalent volts it comes out equal to 2.45 volts, which is in good agreement with the requirements of the Einstein-Richardson formula and is very far from being consistent with Kunz's theory.

Thus we see that we are led to inconsistent and impossible conclusions if we accept Mr. Cornelius' results as a support of Kunz's theory. If, on

[1] O. W. Richardson and K. T. Compton, Phil. Mag., Vol. 24, p. 576 (1912).

[2] Phil. Mag., Vol. 23, p. 264 (1912); Vol. 23, p. 615 (1912).

[3] In this connection Mr. J. W. Woodrow (PHYS. REV., Vol. 35, p. 203 (1912)) in discussing the contact difference of potential between platinum and potassium takes a value from Winkelmann's Handbuch der Physik which is valid only for the metals in contact with electrolytes and is probably not applicable to the case under consideration.

the other hand, we apply these results to test Einstein's and Richardson's theories we find them fairly consistent with the theory and with related facts of thermionic emission and contact difference of potential.

The fact remains however that, while Curve 1 has the better of the argument, it is not in as good accord with the Einstein-Richardson formula as might be expected if this formula is the correct one. In the first place it is not a straight line as the theory demands. In the second place it is quite improbable that there is really any photoelectric emission from potassium at a wave-length as long as 750 $\mu\mu$, which is the longest wave-length used in the measurements under discussion. In fact it is probable that photoelectric emission from potassium takes place only for wave-lengths shorter than about 600 $\mu\mu$. The evidence for this is found in the paper by O. W. Richardson and the writer referred to previously and cannot be gone into fully here. Suffice it to say that the behavior of eight metals, sodium, magnesium, aluminum, zinc, tin, bismuth, copper and platinum indicated that there is a certain critical frequency characteristic of each metal below which there is no photoelectric emission. In the notation already used this critical frequency is accurately given by

$$\frac{300h}{e} n = w_0$$

and is seen, on Einstein's or Richardson's theories, to be that frequency at which the electron acquires just sufficient energy to enable it to escape from the metal. Also results by R. Pohl and P. Pringsheim,[1] who have done very careful work with metals of the alkali group including potassium, bear out this relation, although they were not interested in this particular point. In order to completely remove the argument against the Einstein-Richardson formula it is necessary to account for these two discrepancies.

These variations of the experimental results from the theory might be explained by supposing a thermionic emission superimposed on the photoelectric emission. It is well known that the very electropositive metals like potassium and cæsium with fresh clean surfaces emit electrons in the dark at ordinary room temperatures. It would seem possible that far in the red end of the spectrum where the radiation energy is large, the illuminated surface might be locally heated sufficiently to produce very distinct thermionic currents. A quantitative estimate of the effect possible from this cause may be obtained as follows. In the theory of

[1] Verh. d. D. Phys. Ges., 12, p. 215 (1910); 12, p. 349 (1910); 13, p. 474 (1911); 14, p. 46 (1912).

current-flow in an infinite conductor it has been shown that the resistance offered to the flow of electricity (or heat) from an area on the surface of the infinite conductor into its interior is[1]

$$R = \frac{1}{k}\frac{0.82a}{\pi a^2},$$

where k is the specific conductivity and a is the radius of the area. In our problem we may consider the main body of the potassium to be an infinite conductor with its boundaries at a temperature T_0 and consider the flow of energy across the illuminated area πa^2. If E is the amount of radiant energy incident on unit area of the surface in a unit time and if T is the temperature to which the surface is thereby raised we have obviously in the steady state

$$\frac{dQ}{dt} = \frac{T - T_0}{R} = \frac{T - T_0}{\frac{1}{k}\frac{0.82a}{\pi a^2}} = E\pi a^2,$$

whence

$$T - T_0 = \frac{0.82Ea}{k}$$

gives the rise in temperature at the area of the surface considered. Taking values of a and of E calculated from the probable energy in the arc, the area of the slits in the spectrometer and so on, and neglecting all absorption or reflection by lenses and prism, I calculated the most favorable case I could reasonably construct and found a temperature increase of 5° C. In any actual case it is probably very much smaller than this and this is too small to account adequately for the failure of the curve to conform accurately to the Einstein-Richardson formula.

A much more probable explanation lies in the experimental arrangement described by Mr. Cornelius as follows: "The light [from the arc] passes through a slit and a system of lenses which gives a beam of parallel light upon the prism P, which is capable of rotation, thus providing for an intense source of light, which after passing through the prism, falls upon the slit S of a light-tight box containing the photoelectric cell." The point is that a beam of parallel light strikes the prism and is refracted thence to the photoelectric cell *without being brought to a focus on the slit* in front of the cell. Both in his description and his diagram he describes this arrangement. It is obvious that without the lines of the spectrum being brought to a focus there will be great impurity in the light falling upon the slit. He calibrates his instrument by means of the sodium lines.

[1] Lord Rayleigh, Theory of Sound, Vol. 2, Appendix *A*.

But if the light from the lenses were parallel for the yellow light it would be slightly divergent in the red part of the spectrum unless an achromatic lens system were used to render the light parallel. This divergence, together with the relatively low dispersion in the red spectrum, would still further increase the impurity of the light in the region of the long wave-lengths. The value of the wave-length recorded would correspond to the setting of the instrument and to the average wave-length falling upon the slit. But the maximum potential would be a function only of the shortest wave-length present, which would be quite different from the wave-length recorded. I am not sure but what the experimental arrangement used by Jakob Kunz[1] is not to a smaller extent open to this same criticism. At any rate the error due to impurity in the spectrum would have exactly the effect of changing an Einstein-Richardson curve into one that would more or less fit Kunz's theory, and it seems certain to have played a part in the results of the paper under discussion.

In view of these facts one should not expect more than a very rough agreement between Mr. Cornelius's results and the correct theory and one should expect the agreement to be poorest in the region of long wave-lengths. Taking everything into consideration the agreement with the Einstein-Richardson formula is all that could be expected. But from the evidence given earlier in the paper it is impossible to find in the results of Mr. Cornelius any valid support for the theory of Kunz.

Mr. Cornelius and several others who have been working along this line recently with the alkali metals have been bothered by the fact that frequently a freshly prepared photoelectric cell gives no photoelectric effect until some time after it is made. Then the size of the currents and the maximum potential increase to a maximum and reach a steady state in the course of a few days. The steady state is usually reached more quickly if the cell is heated. This is probably due to a varying contact difference of potential between the surface of the potassium and the platinum receiving electrode. It has been shown[2] that when both the electrodes are earthed there is an effective field between their surfaces which is equal, in the case of potassium and platinum, to about 2.75 volts. The greatest initial kinetic energy found among the electrons liberated from potassium by light in the visible spectrum does not exceed about one volt. Therefore, unless an external field is applied to aid their escape, no electrons will succeed in reaching the receiving electrode. Mr. Cornelius and the others do not seem to have taken this into account. But potassium has a vapor pressure and will slowly distill over on to the

[1] PHYS. REV., Vol. 30, p. 212 (1909).
[2] K. T. Compton, Phil. Mag., Vol. 23, p. 579 (1912).

platinum at ordinary temperatures, there forming an alloy or a surface layer of potassium and thus decreasing the contact difference of potential until the electrons are able to escape, and finally reaching the state where this field between the electrodes has practically vanished. Where an external field has been applied to neutralize the contact difference of potential all the evidence shows that the fresher and cleaner the surface the larger the photoelectric currents.

The relation between frequency, velocity and metal as given by the Einstein-Richardson formula may be most clearly understood by an examination of Fig. 3, which represents all the requirements and implica-

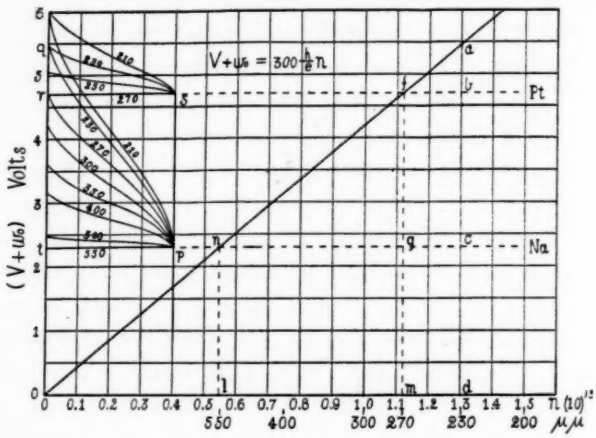


Fig. 3.

tions of the formula. As abscissæ are plotted frequencies, with corresponding wave-lengths indicated below. As ordinates are plotted the values of the total energy, expressed in equivalent volts, acquired by the electrons, which in any case consists of an amount w_0 used in getting away from the metal plus an amount V which represents the initial kinetic energy with which they leave the metal. The diagonal line has a slope $300h/e$. All measurements of electron velocities for any metal or frequency should fall on this line provided the contact difference of potential has been allowed for and the correct value of w_0 is used.

Take for instance the case of platinum for which, as we have seen, we may take $w_0 = 4.7$ volts. The horizontal line marked *Pt* represents $w_0 = 4.7$ volts. Thus for any wave-length, such as 230 $\mu\mu$, the ordinate *ad* represents the total energy acquired by an electron, of which an amount *bd* is lost before the electron gets free of the platinum; whence *ab*, or 0.75 volts, gives the maximum initial kinetic energy with which an electron leaves platinum illuminated with light of wave-length 230 $\mu\mu$.

In a similar manner the electron velocities due to any other wave-length on platinum may at once be predicted. In particular, at a wave-length 270 $\mu\mu$ the energy fm is just sufficient to take the electron from the surface of the metal and no more; for longer wave-lengths no electrons can escape and the metal appears photoelectrically inactive. This point is a strong support of Richardson's theory, since he gets as solutions for his theoretical equations[1]

$$V = \frac{300h}{e} n - w_0 \quad \text{when} \quad w_0 < \frac{300\,h}{e} < \infty$$

and

$$N = 0 \quad \text{when} \quad 0 < \frac{300\,h}{e} n < w_0;$$

showing that the number N of emitted electrons is zero for all values of frequency less that the critical value

$$n = \frac{w_0 e}{300\,h}.$$

Take again the case of sodium. The contact difference of potential between platinum and sodium is 2.4 volts. The line marked Na is drawn at a distance 2.4 volts below the line Pt and its height 2.3 volts above the axis represents w_0 for sodium. We see therefore that the line ac, or 3.15 volts, represents the maximum initial kinetic energy from sodium for a wave-length 230 $\mu\mu$; fg, or 2.4 volts, is the energy for 270 $\mu\mu$; and the critical wave-length is 550 $\mu\mu$. The properties of photoelectric emission from other metals may similarly be represented by drawing corresponding lines, some of which are shown in Fig. 1.

This diagram also fixes the terminal points of the distribution of velocity curves for any given metal and wave-length. As examples I have indicated several of these curves in the upper left-hand corner of the diagram. To be interpreted, rs and tp represent, in the cases of platinum and sodium respectively, the axis along which is plotted the number of emitted electrons; while the distances from r and t respectively to the feet of the various curves represent the maximum initial energies. The location of the foot of each curve is determined from the main part of the diagram. For instance, for platinum and wave-length 230 $\mu\mu$, rq is taken equal to ab, and so on. If Fig. 3 is held with the left side down these distribution of velocity curves appear in their ordinary form.

This figure represents probably as clearly and compactly as can be done the various requirements and implications of the Einstein-Richardson

[1] Phil. Mag., Vol. 24, p. 570 (1912); Science, Vol. 36, p. 57 (1912).

formula. These points have all received quite conclusive support, especially in the work of A. L. Hughes and of O. W. Richardson and the writer mentioned at the beginning of this paper. The predicted values of many of the critical wave-lengths receive additional support in the many papers by R. Pohl and P. Pringsheim[1] and others.

These considerations apply to the relations between frequency, metal and velocity. In addition to giving these relations, Richardson's theory leads to an expression for the number of electrons emitted by any given metal under the influence of a unit amount of light energy of a given wave-length. Experiments on this relation between frequency, metal and number are now in progress at this laboratory. It is interesting to notice that Richardson's theory requires a maximum rate of emission at a wave-length characteristic of each metal and is to this extent supported by the experiments of Pohl and Pringsheim.

It is the hope of the writer that this discussion, by calling attention to various points that must be considered, to the effect of certain systematic errors and to some of the results which have been obtained and the various evidence which supports them, may lead to a clearer understanding of the problem and thence to more accurate results than have hitherto been obtained.

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[1] Op. cit.

THE VARIATION OF THE ALPHA-RAY IONIZATION OF RADIOACTIVE SOLIDS WITH THE THICKNESS OF THE LAYER.

BY HERBERT N. MCCOY.

THE subject indicated by the title of this paper was first studied by Rutherford and McClung,[1] who assumed an exponential absorption law. Later the writer[2] studied the matter extensively after having devised a method of making very thin layers or films of uranium compounds of quite uniform thickness. The results were in satisfactory agreement with an exponential absorption law. After Bragg's discovery that an α particle has a definite range, it became evident that there could be no true "absorption" of the α rays and therefore that an exponential law could not serve as a logical explanation of the facts. Bragg,[3] St. Meyer and v. Schweidler[4] and others have studied the closely related problems of the activity of a thick or thin active layer covered with one or more thin foils.

The work of Bragg and Kleeman[5] and also McClung[6] supplemented by the more recent work of Geiger[7] and Taylor[8] shows the true shape of the ionization curve. Geiger found that the ionization at any point in the path of a particle is inversely proportional to the cube-root of the remainder of its range. This leads to the conclusion[9] that the total ionization of an α particle of range R is proportional to $R^{\frac{3}{2}}$. Although it is very probable that this empirical law is not absolutely exact it is at least a close approximation and as such is very useful in several ways. In a paper just published with C. H. Viol,[10] it was shown that the relative activities of the α-ray products of thorium are very closely proportional to the $\frac{2}{3}$ powers of the ranges of their respective α-rays. I have also found a similar result in a still more recent, unpublished, study of radium and its products.

[1] Phil. Trans., *A*, *196*, 25 (1901).
[2] Jour. Amer. Chem. Soc., *27*, 391 (1905).
[3] Phil. Mag., *11*, 754 (1906).
[4] Wien Sitzungsberichte, *115*, II. *a*, 736 (1906).
[5] Phil. Mag., *8*, 726 (1904); *10*, 318 (1905).
[6] *Ibid.*, *11*, 131 (1906).
[7] Proc. Roy. Soc., *A*, *82*, 486 (1909).
[8] Amer. Journ. Sci., *28*, 357 (1909); Phil. Mag., *21*, 571 (1911); *23*, 670 (1912).
[9] Taylor, Phil. Mag., *21*, 571 (1911).
[10] Phil. Mag., *25*, 333 (1913).

If we assume that the total number of ions produced in air by any α particle is proportional to the $2/3$ power of the distance it travels in air after leaving a radioactive layer, we may readily get an expression for the activity of such a layer as a function of its thickness. Let n = the number of α particles per second from 1 g. of the substance. The number shot upward from 1 sq. cm. of a layer of thickness $dl = \frac{1}{2} n\rho dl$, where ρ is the density. These go equally to every element of a circumscribed hemisphere of surface $2\pi a^2$ (a = radius). Those rays which emerge between the angles θ and $\theta + d\theta$ to a normal strike a zone of area $2\pi a^2 \sin\theta d\theta$ and therefore constitute the fraction $\sin\theta d\theta$ of the rays shot upward; so that the number of rays reaching the zone is $\frac{1}{2} n\rho \sin\theta d\theta dl$. Now suppose the current due to a single α ray is $Cr^{\frac{3}{2}}$ where C is a factor which is constant for the α rays of all substances and r is the effective air range. If R is the range when there is no retardation in the solid, it is readily seen that $(R - kl/\cos\theta) = r$ is the effective range of the emerging ray, if l is the depth from which the α particle comes and k is the ratio of R to the "range" l_0 of the α particle in the solid; that is $k = R/l_0$. The current due to rays between θ and $\theta + d\theta$ from an infinitesimal thickness dl at a depth l from the upper surface is $\frac{1}{2} n\rho C(R - kl/\cos\theta)^{\frac{3}{2}} \sin\theta d\theta dl$ and for the whole of the rays is

$$dA = \tfrac{1}{2} n\rho C \int_0^{\theta} \left(R - \frac{kl}{\cos\theta}\right)^{\frac{3}{2}} \sin\theta d\theta dl,$$

where the upper limit of $\theta = \cos^{-1} kl/R$; since if $\theta > \cos^{-1} kl/R$ the α rays from the lower surface do not get out of the film. If l_0 is the "range" in the solid and $l/l_0 = x$; x is the thickness of a film in terms of the thickness just necessary to give maximum activity. Since $k = R/l_0$; $kl/R = l/l_0 = x$ and therefore

$$dA = Kdx \int_0^{\cos^{-1}x} \left(1 - \frac{x}{\cos\theta}\right)^{\frac{3}{2}} \sin\theta d\theta,$$

where $K = \frac{1}{2} n\rho C R^{\frac{3}{2}} l_0$, a constant for a given substance. Therefore the ionization current of a film of thickness x is

$$A = K \int_0^{x} dx \int_0^{\cos^{-1}x} \left(1 - \frac{x}{\cos\theta}\right)^{\frac{3}{2}} \sin\theta d\theta.$$

If the indicated integral of function θ is written $f(x)$ then

$$A = K \int_0^{x} f(x)dx.$$

As the θ integration gave a very complex expression which presented

difficulties for the x integration, the ordinary method was abandoned and the integrations accomplished without difficulty, by graphic methods. The results are shown in Table I., in which A is expressed in terms of the maximum activity as unity, the numbers in the last column being proportional to those in the third column.

TABLE I.

x	$f(x)$	$\int f(x)dx$	A
0.0	57.30	0.00	0.000
0.1	41.58	4.87	0.284
0.2	31.85	8.52	0.497
0.3	24.06	11.31	0.659
0.4	17.67	13.39	0.780
0.5	12.61	14.90	0.869
0.6	8.44	15.95	0.930
0.7	5.01	16.59	0.967
0.8	2.44	16.96	0.989
0.9	0.76	17.12	0.998
1.0	0.00	17.16	1.000

The graph of A as a function of x is practically identical with an exponential curve up to about $A = 0.5$; while for larger values of x it lies only 1 to 3 per cent. higher. Since my earlier work on uranium compounds and also that of Goettsch[1] had shown agreement with the exponential law within the experimental error of less than 1 per cent., excepting for films of nearly maximum activity where the deviation was a little greater, it follows at once that the facts are also in good agreement with the formula developed above. However the earlier experiments did not conform strictly to the conditions assumed in the derivation of the formula in that the films were made in shallow tins, having rims 0.8 cm. high instead of on flat plates; also, the uranium used contained UX the β rays of which produced a measurable part of the observed activity and finally uranium itself gives two sets of α rays of different ranges.[2]

[1] Jour. Amer. Chem. Soc., *28*, 1540, 1906.

[2] The suggestion that uranium gives two sets of α rays has occasionally been credited to Boltwood, Amer. Jour. Sci., *25*, 298 (1908). I should like to point out that the first announcement of this idea, as far as I can find, was that by Dr. Ross and myself at the Chicago meeting of the Amer. Phys. Soc., Dec. 1, 1906. The abstract of this paper dated November 23, 1906, appeared in the PHYS. REV., January, 1907. In this paper as a result of our measurements of the relative activities of uranium and radium in minerals we concluded that "Since uranium is about twice as active as the equilibrium amount of radium the former must produce two α particles for every one produced by the equilibrium amount of the latter. This indicates that each atom of uranium produces upon disintegration two alpha particles or that there is an alpha-ray product not yet isolated between uranium and uranium X." Dr. Boltwood,

The first two conditions could easily be corrected experimentally, while the effect of two sets of α rays could be treated theoretically.

For an infinitely thin film $f(x) = 57.30$; while for $x = 1$, $\int f(x)dx = 17.16$; Table I. The ratio $\frac{17.16}{57.30} = 0.2994$ is the activity of a film of just sufficient thickness, l_0, to have maximum activity, in terms of the activity of the same mass of material spread into an infinitely thin film on a flat surface. This ratio may be taken practically as 0.3. The activity of a thick film of a uranium compound, say U_3O_8, is the sum of the partial activities of the two sets of α rays. If we call the maximum partial activities of 1 sq. cm. A_1 and A_2 respectively, we may easily find theoretically the relative magnitudes of A_1 and A_2. Let W_1 be the weight of 1 sq. cm. of a film just thick enough to have maximum activity for the U_1 rays (those of uranium itself) and W_2 be the weight for U_2 rays. Let a_1 be the activity due to U_1 rays of 1 g. in an infinitely thin film on a flat plate and a_2 be the activity for the U_2 rays. Then $A_1 = 0.3W_1a_1$ and $A_2 = 0.3W_2a_2$ but $W_1/W_2 = R_1/R_2$ where R_1 and R_2 are the respective ranges and

$$\frac{a_1}{a_2} = \left(\frac{R_1}{R_2}\right)^{\frac{3}{2}}.$$

Therefore

$$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^{\frac{3}{2}}.$$

For uranium Geiger[1] has found $R_1 = 2.37$ and $R_2 = 2.75$ at 0° and 760 mm. Therefore $A_1 = 0.781\ A_2$ and $a_1 = 0.906a_2$; also

$$A_1 + A_2 = 0.3 \times 0.934(a_1 + a_2)W_2.$$

If a film is thick enough to have maximum activity for the more penetrating U_2 rays the activity due to the rays of uranium itself (U_1) should be 0.781 of the activity due to U_2. Maximum activity for U_1 rays would be reached for a thickness $R_1/R_2 = 0.862$ as great as is required for the U_2 rays. By plotting a curve with x and A as coördinates and adding

whose first communication on this subject is in a letter dated December 17, 1906, to Nature, January 3, 1907, wrote: "It will be noted in the above that the activity of uranium is about twice that of the radium present, which is in good agreement with the conclusion of Moore and Schlundt that there are two alpha ray changes in uranium if it is assumed that the average range of the two uranium particles is about 3.5 cm." Dr. Boltwood's conclusion was based, therefore, on the same facts (obtained in independent experiments) as our own. While it is true that Moore and Schlundt did think that UX gives alpha rays, thus suggesting a second alpha-ray change in uranium; this was disproved later and so has nothing to do with the matter in question. See also McCoy and Ross, Jour. Amer. Chem. Soc., *29*, 1708 (1907).

[1] Phil. Mag., *24*, 653 (1912).

to it a curve of the same form, but having its maximum ordinate 0.781 of that of the first curve while its maximum abscissa is 0.862 we get the theoretical curve for the total activity of a film of any uranium compound as a function of its thickness, where the thickness is expressed as a fraction of that required for maximum activity of the U_2 rays. (See Fig. 1.) Table II. gives the total activities for various values of x. It is seen by comparison with Table I. that the total activities thus found are, for each value of x, a little greater than for a substance giving but one set of α rays.

TABLE II.

x	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Activity	0.300	0.522	0.686	0.810	0.894	0.945	0.977	0.993	0.999	1.000

To test the accuracy of these deductions as carefully as possible new experiments were made with films of U_3O_8. Pure uranium nitrate was completely freed from UX and also any possible trace of radium by means of barium sulphate in the usual way. The nitrate was decomposed by heat and the oxide heated to constant weight at 700° in an

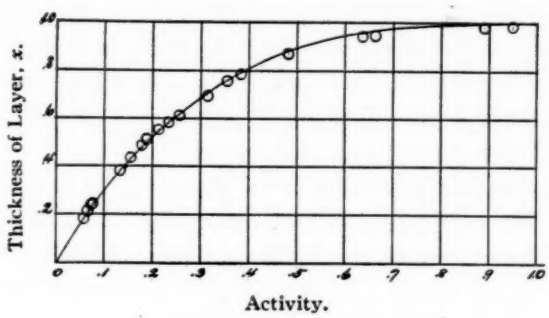

Fig. 1.

Theoretical curve with experimentally determined points.

electric muffle.[1] The product so obtained was pure U_3O_8. This material was made into films on flat circular brass or copper plates in the manner previously described,[2] alcohol being used as the suspending liquid. The films were dried at 105°; those that showed any unevenness or imperfection were discarded. The activity measurements were made in the gold-leaf electroscope described by McCoy and Ashman. The errors of measurement were not over 0.2 to 0.3 per cent.

Two series of films were made; the first on copper plates 7.00 cm. in diameter; the second on brass plates 6.914 cm. in diameter. The results

[1] McCoy and Ashman, Amer. Jour. Sci., *26*, 523 (1908).

[2] McCoy and Ross, Jour. Amer. Chem. Soc., *29*, 1699, 1907.

are given in Table III. and plotted in Fig. 1, where the points are seen to fall very close to the theoretical curve.

TABLE III.

First Series.			Second Series.		
Weight in Mg. per. Sq. Cm.	Observed Activity.	Calculated Activity.	Weight in Mg. per Sq. Cm.	Observed Activity.	Calculated Activity.
0.544	0.179	0.165	0.775	0.240	0.224
0.683	0.217	0.200	0.812	0.243	0.232
1.385	0.392	0.375	1.878	0.485	0.478
1.637	0.437	0.430	1.950	0.498	0.491
1.996	0.512	0.500	2.291	0.569	0.552
2.258	0.551	0.545	3.780	0.766	0.760
2.481	0.590	0.594	5.119	0.867	0.879
2.715	0.612	0.616	9.458	0.980	0.997
3.339	0.693	0.706	10.09	0.985	0.999
3.768	0.759	0.758	12.83	0.991	1.000
4.074	0.783	0.794	13.57	1.000	1.000
6.764	0.941	0.959	16.37	1.000	1.000
7.033	0.946	0.966			
19.87	1.000	1.000			

In order to calculate the theoretical activities it is first necessary to find a factor which converts the weight into the fraction which this weight is of that necessary to give maximum activity. This factor will be a constant for all films of U_3O_8; it may be found from any one determination or the mean of several. This factor proved to be 94; so that $x = 94\ W$ where W is the weight in g. per sq. cm. The values given as "calculated activity" are those corresponding to the value of $x = 94\ W$ indicated by the theoretical curve (Fig. 1). We see that the agreement between facts and theory is quite close; not, perhaps, within the experimental error. But this is not to be expected, as some of the assumptions made in deriving the formula are not absolutely exact. Thus, the assumption that the factor k is a constant for a given substance is not strictly in accord with the fact that the stopping power of a metallic foil is somewhat dependent upon the velocity of the α particle.[1] The error caused by taking k constant is not large and probably would not greatly change the theoretical curve.

By means of an equation given in a foregoing paragraph it is easy to get a second test of the accuracy of the theoretical assumptions made in this paper. It was shown that

$$A_1 + A_2 = 0.3 \times 0.934\ (a_1 + a_2) W_2.$$

[1] Bragg, Phil. Mag., *13*, 507 (1907); Taylor, Amer. Jour. Sci., *28*, 357 (1909); Phil. Mag., *18*, 604 (1909).

If we take the α-ray activity of 1 sq. cm. of a thick film of U_3O_8 as the unit of activity, then $A_1 + A_2 = 1$ and

$$a_1 + a_2 = \frac{1}{0.3 \times 0.934 W_2}$$

$a_1 + a_2$ is the activity that 1 g. of U_3O_8 would have in the form of an infinitely thin film on a flat surface. The value of $a_1 + a_2$ is readily found from the weight and observed activity of films as given in Table III. The ratio of activity to weight increases with the thinness of the film and reaches a maximum for a vanishingly thin film. By a short and quite definite extrapolation of this ratio, as determined by the values given in Table III., it was found that $a_1 + a_2 = 335$.[1] Since $x = 94W$ if $x = 1$, $W_2 = 1/94 = 0.01064$, the weight per sq. cm. just necessary to give the maximum U_2 activity. By substituting this value in the above equation $a_1 + a_2 = 336$. The close agreement between the values found in the two ways is further proof of the correctness of assumptions made in the theoretical discussion.

The "range" of the α rays in U_3O_8 is easily calculated from the above data. If ρ, the density of U_3O_8, is taken as 7.31,[2] l_0, the range of the U_2 rays = 0.01064/7.31 = 0.00145 cm. The range of the U_1 rays is $R_1/R_2 \times 0.00145 = 0.00125$ cm. Then the constant $k = R_2/l_0 = 1890$ for both sets of rays. By substituting for the ranges in air and U_3O_8 the weights per sq. cm., which an α ray can penetrate we get a constant into which a knowledge of the density of U_3O_8 does not enter. The mass of a column of air 1 sq. cm. in section and 2.75 cm. in length is 0.00355 g.; while $W_2 = 0.01064$ g. The ratio of the latter weight to the former is 3.01. This is the ratio of the weights of U_3O_8 and air which produce the same retardation of an α particle; it is the same for both sets of rays of U_3O_8.

The relative "absorption" or, properly speaking, retardation of the α rays in passing through matter of different kinds has been studied extensively. Bragg and Kleeman[3] found that the stopping power of a substance was an atomic property and for equal masses per sq. cm. nearly inversely proportional to the square-root of the atomic weight. I arrived at similar conclusions[4] in a somewhat different way from that

[1] The value, 335, is only half of the total activity, as half of the rays are absorbed in the plate on which the film is made. The total activity is therefore $2 \times 335 = 670$. Since U_3O_8 contains 84.8 per cent. of uranium, this corresponds to an activity of 790 for 1 g. of uranium. This is exactly the value I found in the earliest determination of this constant: Jour. Amer. Chem. Soc., *27*, 391 (1905); while with Dr. Ross, later, *Ibid.*, loc. cit., the value found was 796.

[2] Ebelman, Ann. Chim. phys. (3), *5*, 189 (1842).

[3] Phil. Mag., *10*, 318 (1905).

[4] Jour. Amer. Chem. Soc., *28*, 1555 (1906).

of Bragg and Kleeman. The conclusions reached in the preceding paragraph may easily be compared with the results expected by the square-root law. By this law we calculate that 59.1 per cent. of the retardation that an α particle experiences in passing through a layer of U_3O_8 is caused by the uranium and 40.9 per cent. by the oxygen. Since 0.01064 g. of U_3O_8 containing 84.8 per cent. of uranium, will completely stop a U_2 particle; the same result would be produced by 0.0153 g. of uranium per sq. cm. or by 0.00400 g. of oxygen. The corresponding calculated values for nitrogen and air are 0.00374 g. and 0.00377 g. The ratio of 0.01064 to 0.00377 is 2.82; while the experimental result is 3.01. The calculated ranges in air at 0° and 76 cm. are 2.52 cm. and 2.92 cm. for U_1 and U_2 instead of 2.37 cm. and 2.75 cm. as found by Geiger. The differences are probably largely due to inexactness of the square-root law.

SUMMARY.

1. A formula has been developed for the α-ray activity of a layer of a radio-active solid as a function of its thickness. It was assumed that the air range of an α-ray is diminished in its passage through the solid by an amount directly proportional to the distance it travels in the solid before emerging and that the ionization produced is proportional to the 2/3 power of the distance the α-ray then travels in air. The theoretical curve for the rise of activity with increasing thickness of the layer is almost the same as an exponential curve, although the fundamental equations are entirely different.

2. It has been shown that the activity of a film of any α-ray substance just thick enough to give maximum activity is 0.3 of the activity that the same material would have if spread on a flat surface in an infinitely thin film.

3. It has been shown that the partial activities of a thick film which gives equal numbers of α-rays of ranges R_1 and R_2 are in the ratio $(R_1/R_2)^{\frac{1}{2}}$.

4. It has been found experimentally that the activities of films of U_3O_8 were in good agreement with those calculated by theory.

5. The weights per sq. cm. of U_3O_8 and of air which produce equal retardation of an α-ray are as 3.01 to 1. This ratio is approximately that calculated by the law, first stated by Bragg and Kleeman, that the stopping power of matter is approximately inversely proportional to the square-root of the atomic weight of the material through which an α-ray passes.

In the experimental part of this work I have been ably assisted by Mr. Edwin D. Leman, to whom I wish here to express my sincere thanks.

KENT CHEMICAL LABORATORY,
UNIVERSITY OF CHICAGO, February 10, 1913.

THE PERIODS OF TRANSFORMATION OF URANIUM AND THORIUM.

BY HERBERT N. McCOY.

THE period of uranium has been determined in two ways, first, by counting the scintillations per second from 1 g. of this element and second, by determining the proportion of radium to uranium in minerals, the period of radium being also known. These methods gave 4.8×10^9 and 5.6×10^9 years respectively. A third possibility depends on the linear relation between the logarithms of ranges and decay constants; but this is very inexact as a means of determining periods. In the case of thorium the methods depending on counting the scintillations of the α rays is the only one applicable with any certainty. In the present paper it is shown that the period of a radio-element can be calculated from its range and the ionization current of unit mass. The ranges of all radio-bodies are now known with a fair degree of accuracy; while the ionization currents of uranium and thorium have been carefully measured in connection with other investigations carried out in this laboratory.

It now appears to be well established[1] that the number of ions, N, produced by any α ray of range R is closely proportional to $R^{\frac{3}{2}}$ or $N = aR^{\frac{3}{2}}$. The constant a may be calculated from the work of Geiger and also of Taylor[2] who have determined independently the number of ions formed by an α particle of polonium. The former found 162,000, the latter 164,000. Since $R = 3.58$ cm. for polonium at 0° and 76 cm.,[3]

$$N = 6.97 \times 10^4 R^{\frac{3}{2}}. \tag{1}$$

If I is the total ionization current in amperes of 1 g. of a radioactive element of atomic weight M and decay constant λ, it is easy to show that

$$\lambda \text{ (sec.)} = \frac{IM}{NF} = \frac{IM}{6.97 \times 10^4 R^{\frac{3}{2}} F}, \tag{2}$$

where $F = 96{,}600$ coulombs. If the unit of time is the year

$$\lambda \text{ (year)} = 4.68 \times 10^{-3} \frac{IM}{R^{\frac{3}{2}}}$$

[1] Geiger, Proc. Roy. Soc., A, *82*, 486, 1909. Taylor, Am. Jour. Sci., *28*, 357, 1909, Phil. Mag., *21*, 571, 1911. McCoy and Viol, Phil. Mag. *25*, 333, 1913. McCoy, preceding paper.

[2] Phil. Mag., *23*, 670, 1912.

[3] Geiger, Phil. Mag., *24*, 653, 1912.

and the half-period of decay is

$$T = \frac{\log_e 2}{\lambda} = \frac{148\, R^{\frac{3}{2}}}{IM}. \tag{3}$$

Since the ionization currents or activities of the equilibrium amounts of the members of a given series are proportional to the two thirds powers of the respective ranges,[1] we may also write

$$T = \frac{148 \Sigma R^{\frac{3}{2}}}{I'M}, \tag{4}$$

where I' is the ionization current of 1 g. of the mother element, together with the equilibrium amounts of those products the ranges of which are summed.

The periods of uranium and thorium may be calculated by the aid of the formula 4. The two α rays of uranium have ranges of 2.37 cm. and 2.75 cm. at 0° and 76 cm.[2] and 1 g. of uranium gives an α ray ionization current of 4.61×10^{-10} amperes,[3] therefore

$$T = \frac{148 \times 3.741}{4.61 \times 10^{-10} \times 238.5} = 5.04 \times 10^9 \text{ years.}$$

This result may be compared with that obtained by the scintillation method. The number of particles produced per second was found by Geiger and Rutherford[4] to be 2.37×10^4 for 1 g. of U; taking the value of e as known, the period of uranium was found to be 4.8×10^9 years. Closely agreeing results were obtained by counting the scintillations of a uranium mineral. Brown[5] found only about three fourths as many scintillations as Rutherford, his figures indicating a period of 6×10^9 years. Boltwood and Rutherford's determination[6] of the weight of radium in 1 g. of uranium together with Boltwood's[7] value for the period of radium, 2,000 years, lead to a value of 5.6×10^9 years as the period of uranium. Finally by means of the logarithmic law relating ranges and decay constants,[8] the period of uranium is found to be about 7×10^9 years.

It might be thought that equation (3) which gives the period as a

[1] McCoy and Viol, *loc. cit.*

[2] Geiger, Phil. Mag., *24*, 653, 1912.

[3] McCoy and Ashman, Amer. Jour. Sci., *26*, 528, 1908.

[4] Phil. Mag., *20*, 691, 1910.

[5] Proc. Roy. Soc., A, *84*, 151, 1910.

[6] Amer. Jour. Sci., *22*, 1, 1906. Boltwood, *ibid.*, *25*, 296, 1908.

[7] Amer. Jour. Sci., *25*, 493, 1908.

[8] Rutherford, Phil. Mag., *13*, 110, 1907. Geiger and Nuttall, *ibid.*, *22*, 613, 1911; *23*, 438, 1912.

function of the ionization current and the range would be less useful than one obtainable from the law of Geiger connecting ranges and decay constants, which would give the period as a function of the range alone. But the superiority of equation (3) is apparent if we estimate the effect on the calculated period of a small error in R; by equation (3), a decrease of 0.1 per cent. in R decreases the period by less than 0.1 per cent., while by Geiger's law it doubles the calculated period. However, Geiger's law, in addition to its theoretical significance, is very valuable as a means of estimating the periods of such substances as uranium-two and ionium.

The period of thorium may be calculated from data furnished by earlier work done in this laboratory together with a knowledge of the ranges of thorium and its radio-products. The total activities of 1 g. of uranium + U_2 and 1 g. of thorium + all products are 796 and 1,009 respectively, the unit of activity being that of 1 sq. cm. of a thick film of U_3O_8.[1] Since the total ionization current of 1 g. of uranium + U_2 is 4.61×10^{-10} ampere,[2] that of 1 g. of thorium + products is 5.84×10^{-10} ampere. The ranges of the thorium series have recently been redetermined by Geiger and Nuttall,[3] who give the following for 0° and 76 cm. Th, 2.58; Rt, 3.67; ThX, 4.08; Em, 4.74; A, 5.40; C_1, 8.16; C_2, 4.55. For the whole series $\Sigma R^{\frac{1}{3}} = 16.36$, if but 65 per cent. of $R^{\frac{1}{3}}$ for C_1 and 35 per cent. of $R^{\frac{1}{3}}$ for C_2 be taken in the summation,[4] in accord with the fact that C_1 and C_2 give respectively 65 and 35 per cent. as many α particles per second as each of the other members of the series. By means of the above data the calculated period is found to be 1.78×10^{10} years.

A second determination may be made from the activity of thorium alone, which Ashman[5] found to be 11 per cent. of that of thorium + all its products; this result combined with the range of thorium gives a calculated period of 1.86×10^{10} years.

Geiger and Rutherford[6] found that 1 g. of thorium + its products emit 2.7×10^4 α particles per second and calculate therefrom that the period of thorium is 1.31×10^{10} years. Although there is still a wide difference between the latter value and the one here found, which is about 1.8×10^{10} years, yet both are much smaller than the value 3×10^{10} years which had been assumed earlier.[7]

It is obvious that the period of radium might be calculated by formula

[1] McCoy and Ross, Journ. Amer. Chem. Soc., *29*, 1699, 1710, 1907.
[2] McCoy and Ashman, *loc. cit.*
[3] Phil. Mag., *24*, 653, 1912.
[4] Marsden and Barrett, Proc. Phys. Soc., *24*, 50, 1911. Barratt, Le Rod., *9*, 81, 1912.
[5] Ashman, Amer. Journ. Sci., *27*, 65, 1909.
[6] Geiger and Rutherford, Phil. Mag., *20*, 691, 1910.
[7] See Mme. Curie, Die Radioaktivität II., 544.

(3) if the ionization current of unit mass were known with sufficient exactness. Apparently up to the present time this constant has not been determined directly with much accuracy.

SUMMARY.

The period of a radioactive element can be calculated with considerable accuracy from its range and the ionization current of unit weight. The periods so found of uranium and thorium are 5.0×10^9 and 1.8×10^{10} years respectively.

KENT CHEMICAL LABORATORY,
UNIVERSITY OF CHICAGO,
February 12, 1913.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

ON A PROGRESSIVE DEVELOPMENT OF THE FUNDAMENTAL PRINCIPLES OF MECHANICS.[1]

BY H. M. DADOURIAN.

THE object of this paper is to present in a compact form the salient points of a method of treatment of mechanics which I have followed in a recent book[2] on the subject.

Briefly speaking the following is the general plan of the treatment: (1) The fundamental principle of mechanics is stated in the form of a single law, and in such a manner as to bring it, at least in its simpler aspects, within the experience and mental grasp of the beginner. (2) The interpretation of the law is gradually enlarged as the subject is developed. By this treatment the presentation of the subject is given a high degree of unity and coherence, while its development is made gradual and progressive.

The following is the law upon which the entire subject is based: The sum of all external actions to which a body or a part of a body is subject at any instant vanishes:

$$\Sigma \mathbf{A} = 0.^{3}$$

The meaning of the term 'action,' in other words, the interpretation of the law is widened and developed in four stages, in connection with the application of the law to four different classes of problems.

(*a*) *Equilibrium of a Particle.*—Before applying the law to problems under this head the idea of force is introduced, and the latter is defined as a vector magnitude which represents the action of one particle upon another. Then it is shown that forces are the only type of actions to which a particle is subject, when in equilibrium; consequently the law states that the sum of all the forces to which a particle is subject at any instant vanishes. Therefore the following conditions are obtained for the equilibrium of a particle:

$$\Sigma \mathbf{F} = 0, \quad \text{or} \quad \begin{aligned} \Sigma X &= 0, \\ \Sigma Y &= 0, \\ \Sigma Z &= 0. \end{aligned}$$

[1] Abstract of a paper presented at the New Haven meeting of the Physical Society, March 1, 1913.

[2] Analytical Mechanics, for students of Physics and Engineering, D. Van Nostrand Co., New York, 1913.

[3] Letters in the black type denote vector magnitudes.

(*b*) *Equilibrium of a Rigid Body.*—Here it is shown that a rigid body is capable of having two distinct and independent types of motion, namely, motion of translation and motion of rotation. Consequently it is concluded that a rigid body is capable of being subjected to two types of action, one of which is related to motion of translation, while the other is related to motion of rotation. These are called *linear action* and *angular action*, respectively. Then the law is put in the following form

$$\Sigma(\mathbf{A}_l + \mathbf{A}_a) = 0,$$

where $\mathbf{A}_l$ denotes linear actions and $\mathbf{A}_a$ angular actions. But since $\mathbf{A}_l$ and $\mathbf{A}_a$ are independent of each other the law may be split into the following two sections:

I. The sum of all the linear actions to which a body or a part of a body is subject at any instant vanishes:

$$\Sigma\mathbf{A}_l = 0.$$

II. The sum of all the angular actions to which a body or a part of a body is subject at any instant vanishes:

$$\Sigma\mathbf{A}_a = 0.$$

In the case of equilibrium, forces are the only type of linear actions and torques the only type of angular actions. Consequently the following well-known conditions of equilibrium are obtained from the two sections of the law:

$$\Sigma\mathbf{F} = 0, \quad \text{or} \quad \begin{aligned} \Sigma X &= 0, \\ \Sigma Y &= 0, \\ \Sigma Z &= 0, \end{aligned}$$

and

$$\Sigma\mathbf{G} = 0, \quad \text{or} \quad \begin{aligned} \Sigma G_x &= 0, \\ \Sigma G_y &= 0, \\ \Sigma G_z &= 0. \end{aligned}$$

(*c*) *Motion of Translation.*—Here the concept of *linear kinetic reaction* is introduced as a new type of action which appears whenever a body is given a linear acceleration. The quantitative definitions, or the measures, of kinetic reaction and mass are then illustrated by two ideal experiments. In one of the experiments the particle is supposed to have a longitudinal acceleration while in the other a transverse acceleration.

An accelerated particle, therefore, is subject to two types of linear actions, namely, forces and the kinetic reaction. Hence the law states:

$$\Sigma\mathbf{F}_i + \mathbf{K}_l = 0,$$

or

$$\Sigma\mathbf{F}_i = -\mathbf{K}_l,$$

or

$$\mathbf{F} = m\dot{\mathbf{v}},$$

where $\mathbf{F}$ denotes the sum, or the resultant, of the forces, while $-m\dot{\mathbf{v}}$ is the kinetic reaction. The last relation, which is called the *force-equation*, is then split into two component equations which are related to the tangential and normal directions: Thus

$$F_\tau = m\dot{v}$$

and

$$F_n = -m\frac{v^2}{\rho}.$$

Therefore, in general the magnitude of the resultant force is given by

$$m\sqrt{\dot{v}^2 + \frac{v^4}{\rho^2}}.$$

On this view the "centripetal force" is the real force while the "centrifugal force" is the transverse kinetic reaction. The latter is no more a force than the longitudinal kinetic reaction.[1]

(*d*) *Motion of Rotation.*—Here the *angular kinetic reaction* and the moment of inertia (angular mass) are introduced in a manner similar to the introductions of their linear analogues. Then applying the second section of the law the following relations are obtained:

$$\Sigma\mathbf{G}_i + \mathbf{K}_a = 0,$$

or

$$\Sigma\mathbf{G}_i = -\mathbf{K}_a,$$

or

$$\mathbf{G} = I\dot{\omega},$$

where $\mathbf{G}$ denotes the resultant torque and $-I\dot{\omega}$ the angular kinetic reaction. The last equation is the analogue of the force-equation and is called the torque-equation.

The principles of the conservation of dynamical (kinetic and potential) energy, of the conservation of linear momentum and of the conservation of angular momentum are derived from the law. The following examples show the elegance and ease with which the form of the statement of the law lends itself to proving important dynamical theorems.

Let $\mathbf{F}_i$, $\mathbf{F}_i'$, and $\mathbf{F}_i''$ denote, respectively, the resultant force acting upon a particle in a system, the force due to its connections with the rest of the system, and the force due to external bodies: then applying the law to the particle we obtain

$$\mathbf{F}_i - m_i\dot{\mathbf{v}}_i = 0,$$

or

$$\mathbf{F}_i' + \mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i = 0.$$

Therefore, taking the sum over all the particles of the system we have

$$\Sigma(\mathbf{F}_i' + \mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i) = 0.$$

[1] In order to emphasize the difference between force and kinetic reaction the latter may be defined as the action of the æther (or whatever takes its place in future physical theories) upon a particle of ponderable matter, while force is defined as the action of one particle of ponderable matter upon another.

But, applying the law directly to the system, we get

$$\Sigma(\mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i) = 0.$$

It is evident from the last two equations that

$$\Sigma\mathbf{F}_i' = 0,$$

which states that the sum of the internal forces vanishes.

The principle of the conservation of linear momentum may be proved in the following manner.

$$\Sigma(\mathbf{F}_i'' - m_i\dot{\mathbf{v}}_i) = 0.$$

Therefore

$$\Sigma\mathbf{F}_i'' = \Sigma(m_i\mathbf{v}_i)$$

$$= \frac{d}{dt}\Sigma(m_i\dot{\mathbf{v}}_i).$$

Therefore when the sum of all the external forces acting upon the system vanishes

$$\frac{d}{dt}\Sigma(m_i\mathbf{v}_i) = \Sigma\mathbf{F}_i'' = 0,$$

$$\Sigma(m_i\mathbf{v}_i) = \text{constant}.$$

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THE IONIZATION IN UNSTRIATED DISCHARGE.[1]

BY C. D. CHILD.

A DIFFICULTY which must be met in explaining either the unstriated discharge or the gaseous part of the arc is one concerning the rate of recombination of the positive and negative ions. This rate is proportional to the number of positive ions times that of the negative ones. Both the positive and the negative ions are here proportional to the current, so that the rate of recombination is apparently proportional to the square of the current. Since the rate at which ions are formed must equal the rate at which they recombine, this rate would also appear to be proportional to the square of the current. But there are valid reasons for believing that neither of these conclusions is possible.

This difficulty can be explained, if we remember that there are two kinds of negative ions, one of electronic size and one of molecular size. Approximately all of the current is carried by the first, because of their greater velocity, while a large part of the electrostatic effect is produced by the second kind.

A formula for the relation between current, density of the gas, and the electric force can be derived by making the following assumptions: first, that

[1] Abstract of a paper presented at the Washington meeting of the Physical Society, April 25 and 26, 1913.

the ionization is caused by the impact of the more rapidly moving electrons on the molecules, and secondly that but a small percentage of the collisions between the electrons and the molecules result in combination of the two, while with the great majority of such collisions the electrons and molecules separate after collision without any permanent change in either.

These assumptions lead to the conclusion that the electric force approaches infinity as the density of the gas approaches zero, that it rapidly decreases as the density increases, reaching a minimum when the pressure is a small fraction of a millimeter, and then increases with increasing density. This result agrees with the measurements which have been made on the relation between electric force and density of the gas in these forms of discharge.

This explanation was also found to agree with measurements which were made on the electric force and the temperature and pressure of the vapor in the mercury arc.

By making the further assumption that the light from electric discharge comes from the molecules which have been ionized, this explanation also leads to a formula for the efficiency of the discharge when considered as a source of light. This formula was found to agree with the measurements which have been made on this relation.

On Special Conditions in the Electric Furnace which Produce a Spectrum Similar to that of the Spark.[1]

By Arthur S. King.

THE enhanced lines of metals, a class of lines which appear much stronger in the spark than in the arc, are in general difficult to produce in the electric furnace. However, by operating a tube resistance furnace at a temperature exceeding 2600° C. the writer has recently obtained all of the stronger enhanced lines of titanium from λ 4200 to λ 4600. They are very faint in the furnace as compared to the lines characteristic of the arc spectrum. Special tubes whose walls were made quite thin for a portion of the length gave higher temperatures, with a slight increase in the relative strength of the enhanced lines. Phenomena of special interest appeared when these tubes burned through, forming an arc around the wall of the tube in which 20 K.W. or more were consumed at a potential of about 30 volts. The conditions given by this high-current arc resulted in the enhanced lines appearing very strong, their intensity compared to the arc lines being such as is obtained usually only with a powerful spark.

In order to test whether the condition of the vapor at the center of the tube was different from that near the wall under these circumstances, the spectrum was photographed with a long slit passing across the diameter of the image of

[1] Abstract of a paper presented at the Cleveland meeting of the Physical Society, December 30, 1912.

the tube's interior. The middle of the spectrum line then registered the condition of the vapor at the center of the tube, while the ends of the line were produced by the vapor near the arcing wall.

Photographs made in this manner showed that the arc lines of titanium and the carbon bands were much stronger near the wall of the tube, while the enhanced lines of titanium were as strong in the center of the tube as at the wall, and the spark line of carbon λ 4267 appeared very much stronger in the center than at the wall. This latter line does not appear in the carbon arc burning with continuous current, usually requiring the condensed spark with cold electrodes for its production. The behavior of the arc lines, as well as general considerations, indicate that both temperature and potential gradient are lower in the center of the tube, but it is in this region that the spark lines appear relatively strong in this source.

Another remarkable feature is that the arc lines show a one-sided structure when produced by the vapor at the center of the broken tube, the red side of the line being strongest, while the lines produced near the wall of the tube are nearly or quite symmetrical. This is another point of similarity between the center of the broken tube and the condensed spark, the latter source showing a tendency to give lines apparently displaced toward the red. The regular furnace spectrum taken with long slit, when the tube does not burn through, shows no difference in intensity or structure between center and ends of the line.

MOUNT WILSON SOLAR OBSERVATORY,
December 9, 1912.